vertical phased arrays: part one

Rotatable arrays for the low bands

Forrest Gehrke, K2BT, with two hundred and fifty-two countries worked on 75 meters, has, over the years, followed a natural progression from the use of simple antennas on the low bands to his present 4-square array. This first installment in a multipart series will help dispel some of the myths associated with phased array design. Though many of the statements might at first glance appear obvious, I cannot stress enough the importance of carefully reading this introduction. As Forrest aptly states, a phased array design is not black magic. Achieving outstanding performance just requires a clear understanding of the mechanisms involved. **Ed.**

Many DXers get on the low bands, if they do at all, to fulfill an award requirement. A low inverted-V or dipole is pitched up, the necessary QSLs collected, and then it's back to the HF bands. But some get hooked and stay. They relearn what the radio pioneers discovered: The low bands are a highly predictable and reliable means of long-distance communications, and, in low sunspot periods such as we are now entering, they're the only after-sunset DX game in town. Sorely missing is directional ability, such as even a modest tribander can provide in the HF bands.

Even if it were practical to rotate that low inverted-V or dipole, it would remain a sad fact that most of the signal is radiated at very high angles with virtually no azimuthal directivity. The result is that the impression easily might be gained that the low bands are good for 500 to 1000 mile contacts but no real DX — that is, until the newcomer happens to eavesdrop on one side of a real DX contact. Then he is amazed to hear a Q5 report given, and at the turnover hear nothing except noise. The old adage "You can't work 'em if you can't hear 'em" is particularly apt on the low bands, where atmospheric static as well as manmade noise is very high.

restricting noise pickup

How is it possible to get a low radiation angle and still beat the noise problem? Perhaps this question seems a contradiction because, as the radiation angle is lowered, the paths over which the antenna receives major noise sources are lengthened, whether the noise is manmade or natural. We may not be able to restrict noise pickup in the paths of interest, but we can at least reduce it from undesired paths with a directional array. On the low bands atmospheric noise is very often quite markedly directional, and it is not unusual to find noise levels differing by 30 dB or more between various quadrants of the horizon. Experience shows that high F/B ratio, that is, superior rejection of signals from undesired directions, has far more importance than gain on the low bands for this reason.

It is well known that for reliable DX work a horizontally polarized antenna array had best be one-half to

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two wavelengths above the ground for optimum radiation angle. At 20 meters and shorter this is not too difficult, nor is rotating the antenna, but for 80 or 160 meters such heights become impractical — and rotation is virtually impossible.

One obvious alternative is a vertical antenna with electronic directional control. If such an antenna is combined with a good ground plane, one can get radiation angles as low as those possible with a horizontal antenna two wavelengths above ground. But doesn't a vertical "radiate equally poorly in all directions"? And isn't it said to be noisy? After all, everyone knows that, for some mysterious reason, man-made noise sources are supposed to radiate with vertical polarization. That a vertical's very low radiation angle may have something to do with this is seldom considered.

Widespread misinformation on the vertical antenna in Amateur publications is a serious problem. Recently I researched respected Amateur publications printed since 1970, looking for articles on the vertical that contained definitive technical data. I found only two, one quoting the typical dissimilar and reactive driving impedances of the elements of a two-vertical array,[1] and the other calling attention to the need for maintaining unity current ratio despite this dissimilarity.[2] No quantitative data was available for arrays with more than two elements. A few writers included qualitative comments on the vertical array, indicating awareness of the complexity of the matching situation, but most did not. Perhaps this is because, unlike many horizontal arrays, vertical arrays are often designed with all elements driven, thus making the job of satisfying drive current and phase conditions more complicated.

mutual coupling

At this point it may be useful to review the gain mechanism of a Yagi.[3] The Yagi creates gain in the favored direction as a result of the driving currents and phase currents induced in the parasitic elements by means of mutual coupling between the driven and parasitic elements. With appropriate spacings and lengths chosen for the design frequency, current and phase are caused to exist in each element such that the signal is reinforced in the forward direction and partially cancelled in the other directions. The single driven element will present a significantly lower impedance than it would as a lone dipole, because of the loads coupled to it from the parasitic elements. If a low VSWR is not a goal, this element may be driven directly without affecting the gain pattern of the array. The presence or lack of an impedance transformer (such as a Gamma match) has nothing to do with the gain pattern — only with the match to the feedline. A comparison of the current and phase at the midpoint of each element with respect to the driven element, shows that the current magnitude ratio is below unity (about 0.2 to 0.5.), generally rising or falling in each succeeding parasitic element. The phase angle will lead in the reflector (because this element is longer than a half-wavelength); it will lag at the directors (because they are shorter than a half-wavelength), the angle lagging more in each director as we move toward the front of the array. The interaction is quite complex, since there is mutual coupling among the parasitic elements as well as with the driven element. Nevertheless, it is this phenomenon of mutual coupling that permits us to produce directionality in multi-element arrays.

While it's true that driving each element provides an additional controllable variable, this does not mean that no other drive source is acting on the elements. The same mutual coupling that occurs in the Yagi is present here and must be taken into account as part of the total drive to each element. To illustrate, suppose you want to drive an element of an array with 1 ampere at 90 degrees lagging angle. Assume that, at the same termination impedance of this element, mutual coupling from other elements is inducing 0.8 ampere at 90 degrees lagging. An additional drive current of only 0.2 ampere at 90 degrees lag would be all that's needed. In practice, of course, mutual coupling and this additional drive from the feed network may not add arithmetically. Phase angles probably will be different, resulting in vectorial addition. There's another real life complication: The added drive changes the mutually coupled drive! In fact, changing anything at all changes all the other variables because the mutually coupled elements and feed network are all part of one coupled system. This is why the element driven impedances are referred to as driving-point impedances; they exist only while connected to the feed network. We cannot disconnect any element and verify its value with an impedance bridge.

The assumption that mutual coupling doesn't occur (or isn't important) is a mistake found in many articles on phased arrays, vertical or horizontal, in the Amateur publications. This error is almost invariably compounded by a second and more erroneous one: Electrical length of the delay line is equated to current delay in all circumstances, (for example, a quarter-wavelength line is assumed to produce a 90-degree delay regardless of its termination). But equating electrical length to current delay holds true only under certain conditions:*

1. For any length if terminated by a *pure* resistance equal to the characteristic impedance of the line.

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*Except when specifically noted, only the lossless cases will be considered. At low-band frequencies, losses normally are negligible. Calculations including them add greatly to complexity while resulting in insignificant benefit.

2. For an odd number of quarter-wavelengths if terminated by a *pure* resistance of any value.

3. For any number of half-wavelengths regardless of termination impedance.

4. In some special cases (normally of no concern in these applications).†

Disregarding mutual coupling leads to inaccurate results, particularly as regards front-to-back ratio. The designer who makes this error is also typically led to some or all of the following subsidiary assumptions:

1. That the driven impedances of each element always are equal.

2. That if the elements are resonant, the driven impedance of each element is resistive.

3. That if array feedlines are quarter-wavelength, a 90 degree phase change in current is produced in each line.

4. That if the array requires equal current drive, driving each element with equal power will always satisfy the requirement.

5. That a current phase angle displacement of 90 degrees between array elements will occur by insertion of a quarter-wavelength line in the feedline of one of the elements.

Every one of these assumptions is **wrong**, because the premise on which they are based is not true.

Some writers suggest that great liberties may be taken with element feedline lengths. Without considering the effects upon phasing, they would use element feedlines of any length as long as they were equal. Except in very specific circumstances (when all driving impedances are equal), there is no way to justify taking these liberties with most multi-element array configurations.

array impedances and power distribution

It may be illuminating to examine a typical set of dynamic driven impedances for the quarter-wave resonant elements of a 4-square vertical phased array (fed with equal-magnitude currents of the proper phases to produce the main lobe along a diagonal). This will demonstrate the profound effects of mutual coupling.

element 1	$Z_1 = 7.9 - j7.8$
element 2 or 3	$Z_2 = Z_3 = 35.7 - j12.7$
element 4	$Z_4 = 59.2 + j42.6$

The first impedance is the reference, or zero-degree phased element; the next is the impedance of each of the two −90 degree phased middle elements; the last is the −180 degree phased element. That these impedances are quite dissimilar and reactive is obvious. Since drive power is a linear function of the *real* component of these impedances (being fed with currents of equal magnitude), it is clear that power division among these elements is far from equal. Assuming 1-ampere drive to each element, the drive power supplied to each is:

element 1	7.9 watts
element 2	35.7 watts
element 3	35.7 watts
element 4	59.2 watts

which, on a percentage basis, is 5.7 percent, 25.8 percent, 25.8 percent, and 42.7 percent, respectively. Thus a feed network aimed at supplying equal power to this array, such as a Wilkenson power divider, will be at cross purposes with the requirement. (Incidentally, a Wilkenson divider will *not* supply equal power to unequal terminations.) Also, since the 90-degree phased elements are not resistive, simply inserting a quarter-wavelength of delay line in their feeders won't do. Clearly, only a feed system designed for the array elements' driving-point impedances will carry out this unequal power division while producing the proper element phase displacements.

It is possible to devise a feed network which performs these functions while also matching the array to the transmitter feedline. Doing so is not even unduly complex, but calculating the driven impedances does require a knowledge of the self and mutual impedances of the elements. Methods for doing this will be detailed in a future article. The greatest benefit of a good match in multi-element arrays is the warning it provides when loss of continuity to an element occurs because of faulty switching relays or the like.

30 to 40 dB F/B are achievable

My interest in low-band DX began just as described in the beginning of this article. I started with a dipole 30 feet high, then progressed to a vertical, and then to in-line arrays of two and three verticals. With some cut-and-try, the arrays were made to work quite well.

Then came the articles by W1CF on the 4-square

†Special cases are mentioned for completeness. The situations governing them are not ordinarily encountered in phased-array feed network applications. These cases arise when the real and reactive components of a termination have a particular relationship with the characteristic impedance, Z_O, of the line and its electrical length. For example, an eighth-wavelength line will have a current delay of 45 degrees with terminated by an impedance whose arithmetic sum of the real and reactive components equals Z_O. A three-eighths wavelength line, under the same impedance relationships, will exhibit 135 degrees phase delay between input voltage and output current. These are two special cases which I explored; there may be more. I am indebted to W7EL for bringing the possibility of such unusual cases to my attention.

array[4] which inspired me, as they have many others, to duplicate his pathfinding work in building pattern controlled low-band arrays. For me at least, having achieved excellent F/B with simpler arrays (but without bothering to find out precisely why), the F/B results were disappointing. Cut-and-try led nowhere, this array's having too many variables for such blind stabs, and so I had to go back to basics for a more fundamental understanding. Thanks to the advice, encouragement, ideas, and boundless resource of mathematical tools contributed by my friend WB6SXV, as well as many information exchanges with W7EL and W2PV[5] I believe I now know how the 4-square should work.

Achieving theoretical F/B in practice ultimately becomes an exercise in achieving electrical symmetry of the array. This is not easy, but efforts continue to reach that goal. Fortunately, like Yagis, these arrays *want* to work. Less than optimum drive conditions for forward gain find them as tolerant as Yagis, but also as intolerant for high front-to-back ratio. Despite large departures from design drive currents and delay angles, forward gain is not affected much. But seemingly insignificant differences in drive currents or delay angles drastically reduce the maximum F/B capabilities. A 10 percent change in drive current of one element in a 4-square can bring the array from a really excellent 30 to 40 dB F/B down to an average 15 to 20 dB. Another way of looking at this is that excellent F/B ratios hold over a small frequency range, while gain holds over a relatively much larger range, as W2PV showed for the Yagi.[3]

Although the principles for correctly feeding a multiple driven element array have been known since the 1930s,[6,7] their primary application has been by the long-wave a-m broadcast industry, and relatively little has been published in Amateur Radio literature. Perhaps editors may have felt the subject too complex, or that it lacked broad reader interest. Another possible reason is that few modern antenna texts discuss feed methods for such arrays. Typically, many field plots are shown, but means for achieving them are left to the reader.

areas to be addressed

It is the purpose of this series of articles to attempt to fill this gap. Over the next few months I shall try to address the following considerations:

I. Theoretical Array Design
  - Element spacing
  - Drive requirements — magnitude and phase
  - Field plotting — how to calculate

II. Self and Mutual Impedance
  - Measurements and calculations
  - Ground planes
  - Element driven impedances

III. Drive Network Design
  - Four-terminal network matrices
  - Pi and T coax equivalents
  - Directional switching
  - Adjustment and measurement

Topics of this nature cannot be adequately discussed without presenting voltages, currents, and impedances in complex algebraic form, such as R + jX for impedance. Those readers who understand them will have no difficulty in following the presentation; for those who do not, I am assuming that they have a good enough general understanding of the concepts (of resistance and reactance) to be able to understand the implications of the conclusions I present.

In general, I shall try to address myself to general solutions, without restriction to specific designs. Where particular designs are examined, these will be by way of illustration, not for the sake of presenting any one proposal. Rather, it is my hope that readers will find their own solutions to their particular problems within the space they have available. There is nothing writ in stone, for example, which requires the elements of an array to be resonant, to be spaced at 1/4 wavelength, to be phased in multiples of 90 degrees, or to have radials measured to some exact length. Neither do all arrays operate best with equal current magnitude to all elements. A few hours of mathematical experimentation will allow you to run through more designs than you could ever hope to build.

Building vertical phased arrays is not a black art; with accurate measurements of self and mutual impedances and with reasonably good electrical symmetry, theoretical design goals can be closely approximated in practice. Most of the explanation for the large gap between theory and practice which so many builders encounter lies in the many invalid assumptions discussed earlier.

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3. James L. Lawson, W2PV, "Yagi Antenna Design," *ham radio*, January, 1980, page 22; May, 1980, page 18; June, 1980, page 33; July, 1980, page 18.
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ham radio

vertical phased arrays: part 2

Part two examines array siting, field plot calculations, and minor lobe determinations

The theoretical design of vertical phased arrays will be the subject of this article, the second in this series on vertical antennas. Every designer must balance his performance requirements for high gain and F/B (front-to-back ratio) against his resources (space, money, and time). There is always a strong temptation to skip past the theoretical work and proceed with the more engrossing task of construction. But after having relocated the elements of an array (complete with one-hundred radial ground systems) more than once, I can tell you that a few thoughtful hours spent on design can save many hours of wasted construction time.

For example, you might want to start with a two-element array. But before you clear a site of brush and trees, consider what you might be faced with should you later decide to add elements to this array. And consider the directions of the main lobe of the changed array. For instance, a two-element array has a main lobe whose half-power beamwidth is 180 degrees. Before you decide to aim this array toward Europe or Japan remember that the signal loss sustained in orienting this array exactly east and west is about 1/2 dB (down from Europe or Japan) and only 3 dB — half an S-unit — down in a north or south direction (see **figs. 3** and **4**)*. I live in a wooded area and, not being willing to become involved in a lum-

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*Charts as introduced in the text are not in sequence. However, they are grouped as follows: two-element arrays, **figs. 3** and **4**; three-element arrays, **figs. 5** through **11**; four-element arrays, **figs. 12** through **17**. **Editor.**

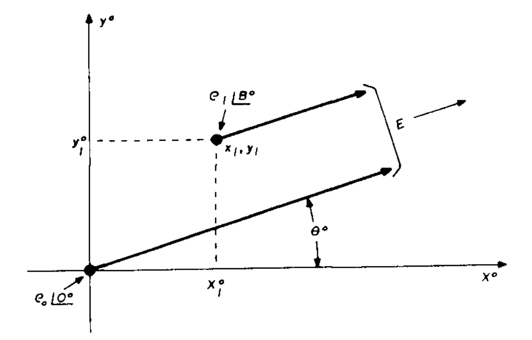

fig. 1. Generation of cumulative field in the θ direction by the two sources $e_0 \underline{|0}$ and $e_1 \underline{|B}$.

bering operation, I had to strategically locate a 4-square array among the trees. Because I failed to give some advance thought to these considerations, relocations of elements were required with every addition to my array.

In a previous article,[1] I discussed some of the reasons for less-than-anticipated performance in vertical phased arrays, particularly in front-to-back ratio. The major fault in many designs is a failure to consider the real and significant effects of mutual coupling between elements; neglecting these terms can result in an incorrectly designed feed network.

symmetry is essential

Knowing the characteristics of the array, we can ensure that the correct current magnitude and phase exist at the input terminals of each element — for a particular direction.

However, in a switchable array the magnitude and phase drive current requirements to a specific element will depend on position (direction) chosen. Conversely each element must perform correctly for each switched direction of the array. If we expect to have identical field patterns in each direction it is necessary to ensure electrical symmetry. For example, referring to the driving-point element impedances of a typical 4-square array, spaced 1/4 wavelength, with equal amplitude current ratios, phased −90 and −180 degrees we have:

$$Z_1 = 7.9 - j7.8$$
$$Z_2 = Z_3 = 35.7 - j12.7$$
$$Z_4 = 59.2 + j42.6$$

The zero degree phase reference elements driving point impedance is Z_1. The diagonally opposite element is phased −180 degrees and its driving point impedance is Z_4. The two middle elements, each phased −90 degrees present the same impedances Z_2 and Z_3. When this array is switched through its four directions, each element, in turn, assumes each of the four electrical positions in the array. For identical patterns in each direction each element must exhibit the particular driving point impedances appropriate to these electrical positions. For example, as each element is switched to occupy electrical position 4, the same driving point impedance Z_4 must be presented to the feed network. Similarly, as each element is switched into electrical position 1 it must present the much different impedance Z_1. So, instead of physically rotating this antenna, keeping each element fixed in its electrical relationship as with Yagis, we rotate the *electrical* relationship of the elements and keep the *physical* relationship fixed.

This is an important difference from the design of a-m broadcast arrays. Broadcast arrays are seldom switched, being designed for a particular listening area, and with departures from symmetry often intentional. For a switched array, each element's self-impedance, and each of its mutual impedances, must be as similar as possible.

Electrical symmetry is a function of the physical symmetry of the array which includes groundplanes and other nearby conductive structures. Metal towers, other antennas, guy wires, roof gutters, and leaders — that is, any conductive line within a wavelength of any part of the array, especially if it is at or near resonance (a multiple of a half-wavelength and ungrounded, or a quarter-wavelength and grounded) — should be avoided. Otherwise, a means must be found to prevent resonance. An example of preventing resonance would be to break up guy wires with insulators, making them ungrounded quarter-wave sections. When siting an array, then, look carefully around the area before starting work for anything that can act as another antenna. Unlike Yagis, where making spacing adjustments involves loosening a few clamps, low-band vertical arrays, with their groundplanes, are not easy to make adjustments on.

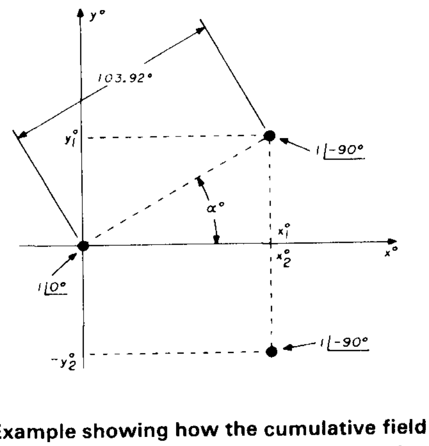

fig. 2. Example showing how the cumulative field from an equilateral triangular array can be calculated.

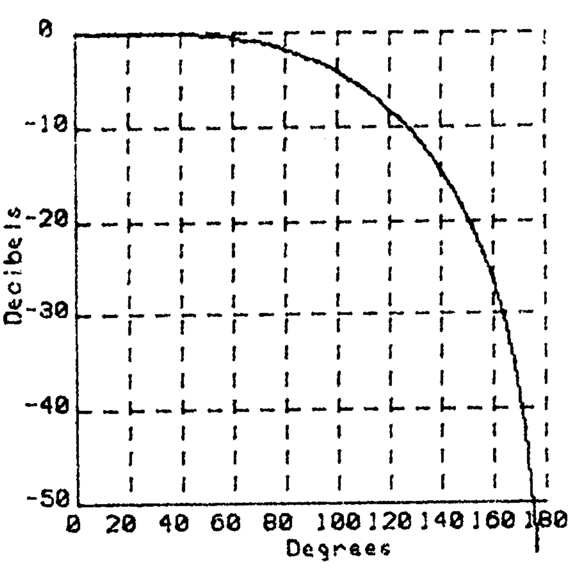

fig. 3. Two-element, 1/4-wave spacing −90 degree phasing, equal amplitude current array. [In figs. 3-17, 0 through 180° shown. *Editor*]

extensive ground systems

Don't scrimp on the groundplane. At least sixty radials a quarter-wavelength or longer should be under each element. If in some directions this is not possible, use radials at least an eighth wavelength long in even larger quantity. At those azimuths your array will probably have a higher elevation angle. However, more radials, even if short, help keep the angle down.

Theoretically, an infinitely conducting groundplane is required, but it's not practical to copper plate the neighborhood. Don't make the mistake of thinking that twenty or thirty radials is approaching the point of overdoing it! Incidentally, if you can, lay radials on the surface. If you must bury them, keep them as close to the surface as possible. Large-size wire is not necessary; I use No. 24 PVC hookup wire. Galvanized steel fence wire is not a good idea: it corrodes very quickly, becoming totally ineffective as a radial.

characterizing the array

After choosing the site it's necessary to see what kind of an array can be fitted within that area, what its characteristics might be and its switchable directions. The calculation of horizontal field patterns is an exercise in trigonometry.

Since F/B ratio is a major interest, we need to explore in greater detail the pattern in the rear area of the array. Inspection of the field equation shows that subtractive operations take place here, often resulting in small fields which have large variations with little changes in azimuth. This necessitates many azimuth calculations to reduce the granularity of the plot. Compare the similar arrays of **fig. 13**, a plot in 2-degree increments, with **fig. 16**, plotted in 10-degree increments. We would not want to miss seeing the actual variations, since deep nulls may be used later in checking out the array. A programmable calculator or a small computer is an obvious choice for handling this drudgery. When the HP-35 scientific calculator was introduced 10 years ago I plotted a three-element in-line array using 10-degree increments. The process required about 8 hours and seemed lightning-fast, but everything is relative; now I watch a Sharp PC-1500 do this in 2-degree increments in 5 minutes — including drawing a graphical representation.

multi-element array equation

The equation for the total field from any multiple element antenna is:

$$* \; E = e_0 \underline{/B_0^\circ + X_0 \cos\theta^\circ + Y_0 \sin\theta^\circ} + \ldots + e_n \underline{/B_n^\circ + X_n \cos\theta^\circ + Y_n \sin\theta^\circ}$$

*Choosing a common unit of dimension (e.g. degrees) for the vector terms allows simplification of the programming task.

where E is the total field term

$e_0, \ldots e_n$ are the individual term amplitudes

$B_0, \ldots B_n$ are the driving-point phase displacements with respect to the reference term

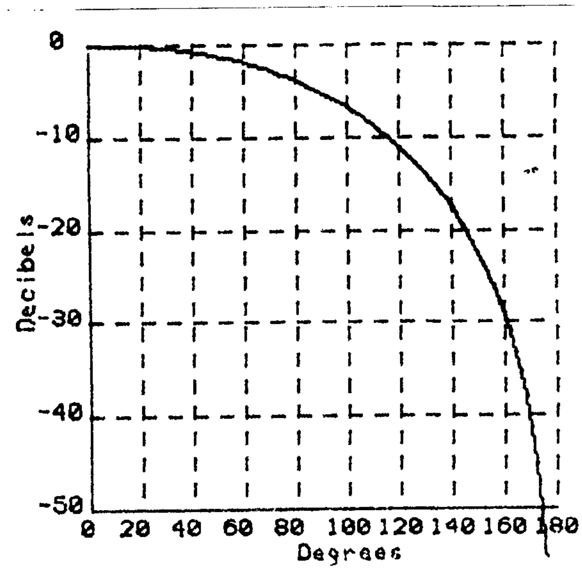

fig. 4. Two-element, 1/8-wave spacing −135 degree phasing equal amplitude current array.

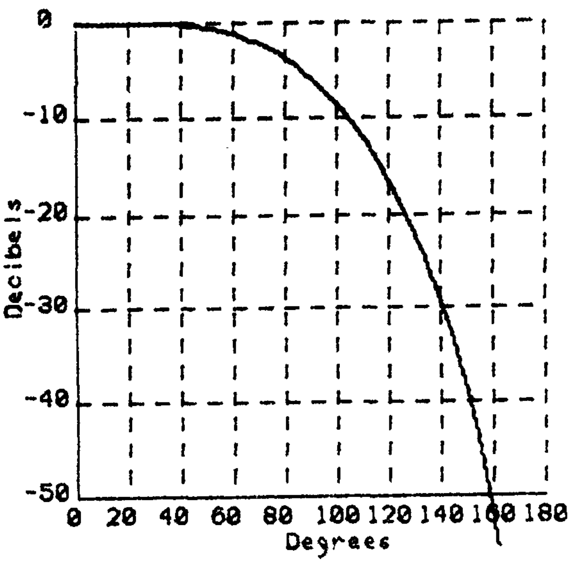

fig. 5. Three-element, in-line configuration, 1/4-wave spacing, −90 degree and −180 degree phasing, 1:2:1 current ratios.

$X_0, \ldots X_n$ and $Y_0, \ldots Y_n$ are the physical distances in terms of degrees of wavelength from the $0,0$ coordinates

θ = horizontal direction considered

There will be as many terms as there are elements. It is usually convenient to place one of the elements at the X-Y axis (origin) and to consider this element the reference element. This also simplifies calculations since all of its angular components then equate to zero. Since we are interested only in the magnitude of the vector sum of the individual terms, the angle resulting from this calculation is discarded.

two-element array calculations

Referring to **fig. 1**, consider a two-element array, with the reference element located at the origin. e_0 is the amplitude of the electric field of this element at some given distance in any direction, with a drive phase displacement B_0 of 0 degrees. Similarly, at the same distance, the field of the other element is e_1 with its driving-point phase displacement of B_1 degrees with respect to the reference element. At the given distance (assumed to be far enough removed from the array so that the combined field can be considered a plane wave) E is the vector sum of the fields e_0 and e_1 in the horizontal direction θ degrees. Note that both displacements, the physical and the electrical terms, are given in degrees.

We are interested only in determining a relative field plot for an array. We want to know what the fields are at various azimuths relative to the field at some fixed angle (usually chosen as the maximum field direction). Provided all the elements are identical, we can substitute current for voltage and we can state this current as a ratio of the reference-element current amplitude. For example, if each element were to be fed with equal current amplitude, the ratio would be 1 for each element.

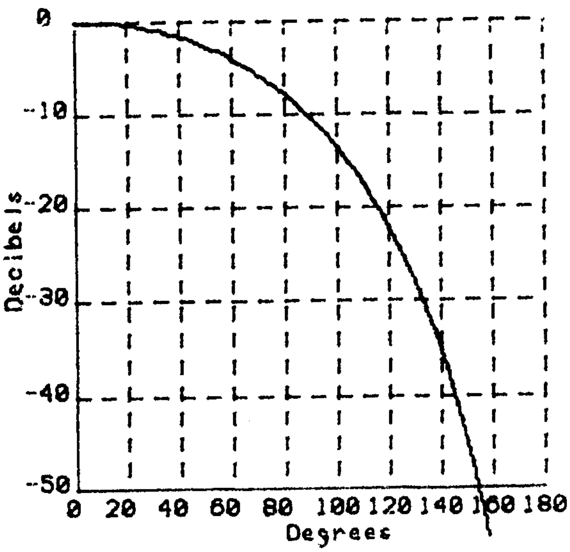

fig. 6. Three-element, in-line configuration, 1/8-wave spacing −135 degree and −270 degree phasing, 1:2:1 current ratios.

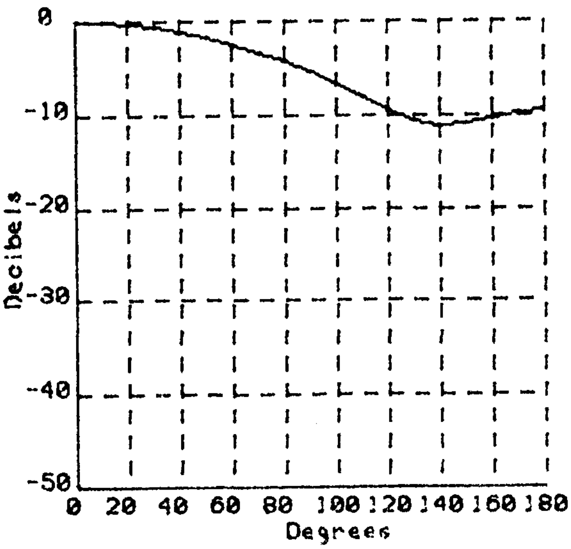

fig. 7. Three-element, triangular configuration, 0.289 wave spacing −90 degree and −90 degree phasing, 1:1:1 current ratios.

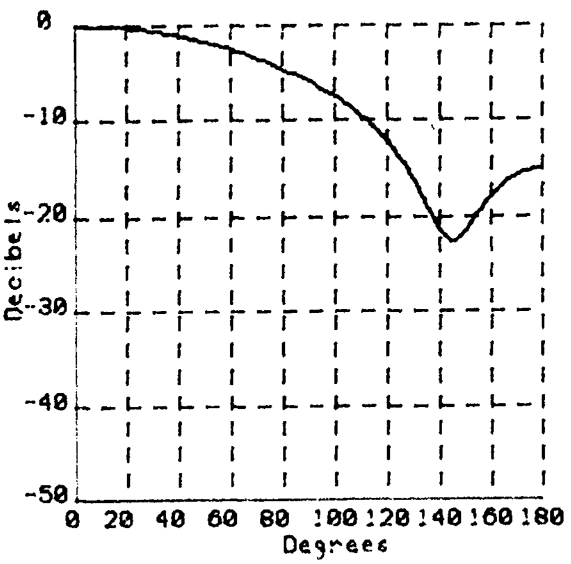

fig. 8. Three-element, triangular configuration 0.289 wave spacing −110 degree and −110 degree phasing, 1:0.5:0.5 current ratios.

three-element array calculations

A specific example illustrates how to use this equation. Referring to **fig. 2**, assume an equilateral triangular array with 0.289 wavelength spacing (that is, 103.92 degrees):

Since this is an equilateral triangle, $\alpha = 30\ degrees$.

$$X_1 = X_2 = 103.923 \cos 30^\circ = 90^\circ$$

$$Y_1 = 90 \tan 30^\circ = 51.962^\circ$$

$$Y_2 = 90 \tan(-30^\circ) = -51.962^\circ$$

If equal amplitude current drive feeds the array and elements 2 and 3 are both phased −90 degrees, the field at any azimuth θ degrees is:

$$I = 1\,/0^\circ + 1\,/-90^\circ + 90 \cos\theta^\circ + 51.962 \sin\theta^\circ + 1\,/-90^\circ + 90\cos\theta^\circ - 51.962 \sin\theta^\circ$$

Substituting values for θ°, we get:

θ°	I (relative current magnitude)
0	3.00
30	2.78
60	2.24
90	1.59
120	1.00
150	0.85
180	1.00

Refer to **fig. 7** for a relative power plot of this array.

This graph requires explanation, since I have further manipulated the results for the portrayal of this data. First, the results are normalized, by dividing each result by the maximum value. Second, the logarithm (base ten) is taken of each normalized value and multiplied by 20 to make all the calculated points relative to 0 dB. Thus:

$$dB\ (at\ azimuth\ \theta^\circ) = 20 \log_{10} I/I_{max}$$

Since the maximum value for I occurs at 0° azimuth, then normalizing to this value in terms of dB for the data listed above:

θ°	decibels
0	+0
30	−0.65
60	−2.55
90	−5.53
120	−9.54
150	−10.99
180	−9.54

This method of representation best displays array rejection capabilities, not easily shown in a polar plot. For example, assume an array with a respectable −30 dB F/B ratio. Whatever scale is used for the direction of maximum signal must now be divided by 1000 to show this rejection. This will appear as little more than a flyspeck on a polar plot and provide no clear indication of variation with azimuth. Suppose we are listening to an S 9 + 30 dB signal at the front of our array; switching the array around, to the rear

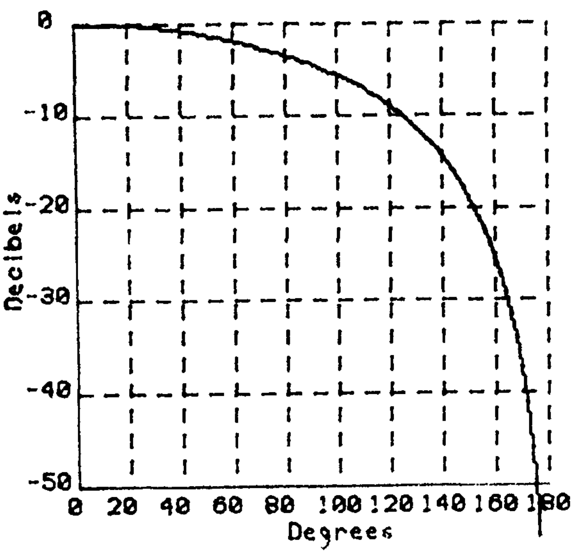

fig. 9. Three-element, triangular configuration, 0.289 wave spacing −90 degree and −90 degree phasing, 1:0.5:0.5 current ratios.

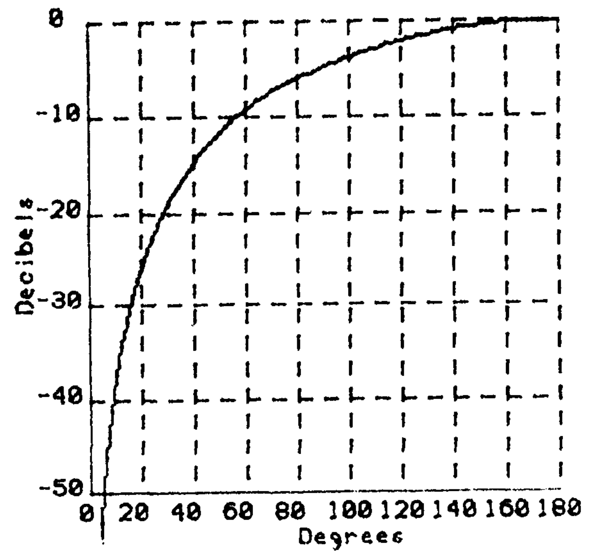

fig. 10. Three-element, triangular configuration, +90 degree and +90 degree phasing, 1:0.5:0.5 current ratios.

we will still see S 9, a not insignificant signal. Yet the reduction is by a factor of 1000, and if the transmitter is running a kilowatt, our array will treat it as though it were only one watt! This illustrates the need for working with logarithmic decibels. But we should not forget what they represent; they are not linear.

If the array is symmetrical and has been located symmetrically about the x or y axis, it is not necessary to plot more than 180 degrees; the other half is a mirror image.

determining array gain

The value for I, in the direction of maximum signal, is not an *absolute* gain figure. This value is merely derived from the number of elements and the absolute current ratios used. An indication of gain is obtained by observing the total included angle of the main lobe between the half-power (-3 dB) points; thus the smaller the included angle the higher the gain. The simplest way to determine gain is to make a polar power plot (square each azimuth calculation result). Calculate the area of this lobe and then determine the equivalent radius of a circle having the same area. Using the same scale, the ratio of the length of the maximum lobe vector to this equivalent radius is the gain of the array over a single vertical element. However, on the low bands F/B ratio is much more important than gain. For the purposes of making vertical array evaluations from the field equation, keep in mind these implied assumptions:

1. There is an infinitely conducting groundplane.
2. All elements are electrically identical.

Any departure from an infinitely conductive groundplane results in lower efficiency due to ground losses and a higher vertical radiation angle (of maximum signal). If the elements are not electrically identical, the real field pattern differs from the calculated one. For switchable arrays using the same feed network, further complications occur. Even the real field patterns will not be alike.

n-element array calculations

Using the field plotting equation and a programmable calculator, any array layout can be examined. Simply choose the angular coordinates for each element, their drive current amplitude ratios and phase displacements. There are no restraints in choices of current amplitude ratios and phase displacements. (Later, these values will be used in calculating the element driving-point impedances, which in turn will determine the feed network.)

Experimentation shows that equal current to all elements is not always best; neither are element spacings of 1/4 wavelength or current phase displacements of 90 degrees always optimum. **Fig. 11**, an equilateral triangle array, best illustrates these points. This array has elements spaced 1/8 wavelength apart with two of its elements operated at a current amplitude ratio of 0.5 and current phase dis-

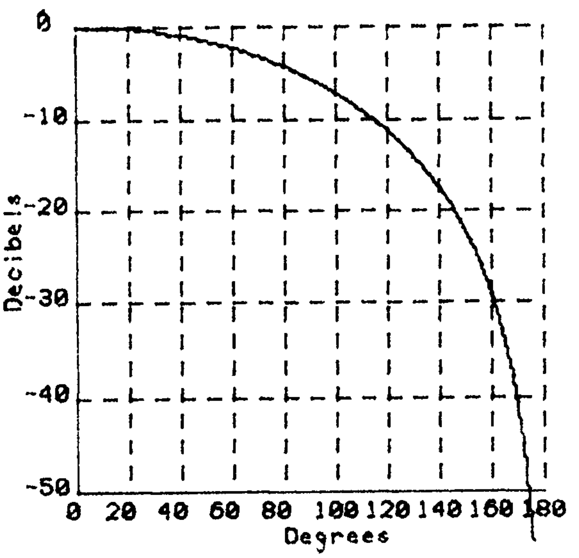

fig. 11. Three-element, triangular configuration, 0.144-wave spacing −135 degree and −135 degree phasing, 1:0.5:0.5 current ratios.

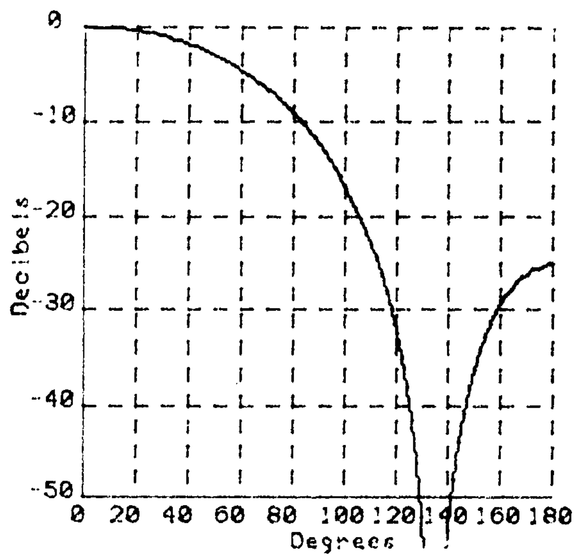

fig. 12. Four-element, 4-square configuration, 1/4-wave spacing −90 degree and −180 degree phasing, 1:1:1:1 current ratios.

placement of −135 degrees. A number of representative arrays have been plotted to show their general properties and to illustrate the variations that occur with changes in physical layout or varying input drive conditions.

two-element arrays

Figs. 3 and **4** are two-element array plots. These produce cardioid field patterns when driven with equal amplitude current with a phase displacement of −90 degrees. The half-power beamwidth is about 180 degrees with a theoretically infinite F/B at precisely 180 degrees azimuth. The **1/8** wavelength spaced array has a slight edge in F/B performance but because of close spacing it has some special problems of its own which I will discuss presently.

The **1/4**-wavelength version is quite tolerant of drive condition deviations and displays a useful F/B ratio even with phase displacements far from optimum. These characteristics, plus its simplicity and small space requirement, account for its popularity. The easy tolerance of this design may also account for the unwarranted but widespread belief that more complex arrays will be equally amenable. It so happens that if the feed network of this array consists of 50-ohm, 1/4-wavelength coaxial feeders to the elements and a 1/4-wavelength delay line, nearly optimum drive conditions will exist for best F/B.

triangular arrays

The triangular array plots aptly illustrate design parameter variation (**figs. 7**, **8**, and **9**). Prior articles[3,4] proposed 0.289-wavelength element spacing (resulting in 0.25 wavelength for the distance from apex to base of the triangle). W1CF proposed unity current ratios, with two of the elements phased −90 degrees (**fig. 7**). W2PV proposed phasing these two elements at −110 degrees and reducing their current amplitude ratios to 0.5 (**fig. 8**). The result for both of these arrays, while providing three alternatives for beam direction, is a not very spectacular maximum F/B ratio of −10 and −15 dB, respectively. If parts of both proposals are combined, that is, phases of −90 degrees and current ratios of 0.5 for these two elements, maximum F/B ratio improves to nearly infinity (**fig. 9**). A little time spent with a calculator has changed a not-too-interesting array into an exciting performer.

Both writers proposed to make these arrays switchable in six directions; W2PV omitted explaining how, and W1CF proposed an equal power divider. However, since the elements will not present equal and resistive driving-point impedances (in any of these variations), W1CF's intended field plot cannot be achieved with equal power division. Since the half-power beamwidth of these arrays is about 135 degrees, the additional complexity required to switch

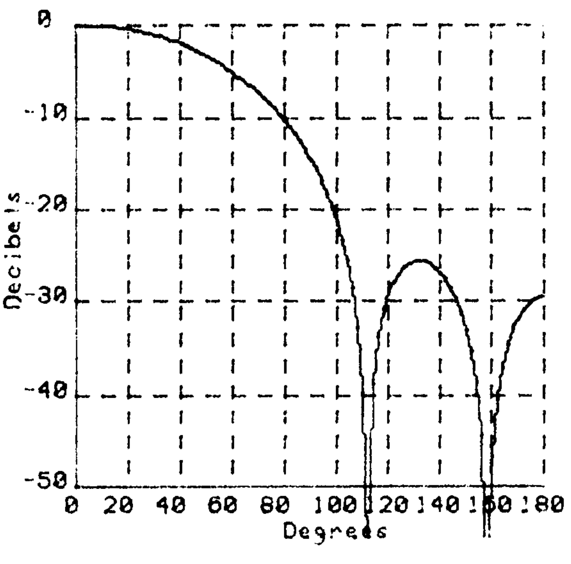

fig. 13. Four-element, 4-square configuration, 0.272-wave spacing −90 degree and −180 degree phasing, 1:1:1:1 current ratios.

this array in six directions makes it of questionable value. Nevertheless, we can develop a feed network which produces a leading phase of +90 degrees, making the array switchable in six directions (see **fig. 10**). Two feed networks are required. A future article will present a lumped constant network equivalent to coaxial lines, except that current phase may be advanced as well as delayed (and may be designed for any characteristic impedance one happens to require!).

three-element in-line arrays

This antenna is another example of an unequal current-amplitude-ratio driven array. The middle element current is twice that of the reference element. The 1/4-wavelength spaced version, properly driven, has a 90 degree included angle over which the F/B is better than −25 dB (approaching infinity at the 180 degree azimuth). The half-power beamwidth is about 150 degrees and is down −6 dB, or one S-unit, at the ±90 degree azimuths. On 80 meters the F/B capability of this array has been impressive in listening tests, even with nearby stations (within 20 miles), the ultimate test of F/B on the low bands. I have often considered the possibilities of an antenna consisting of two such arrays, operated at right angles to each other.

The 1/8-wavelength spaced array plot is a near duplicate of the wider spaced array. It has slightly higher gain as a result of its narrower half-power beamwidth of 110 degrees and it has a wider width over which F/B exceeds −25 dB.

4-square arrays

These arrays, although having four elements, are closely related to the three-element in-line type. The array projects its main lobe along a diagonal of the square, with the two middle elements driven at the same phase and the current divided between them. In effect, the middle element is split into two elements. Half-power beamwidth is about 95 degrees, indicating a gain increase over the three-element in-line. The width over which the F/B is −25 dB or better has increased to 150 degrees, though the average rejection over this range is not as deep as with three-element in-line arrays. The symmetry of this element arrangement allows the array to be switched in four directions using the same feed network; that is, the main lobe may be formed in either direction along either diagonal. As has been pointed out earlier, due to the significant dissimilarity of drive-point impedances of any element as the array direction is switched, more than ordinary care must be taken to ensure electrical symmetry. Experiments with the field equation demonstrates the high minor lobe sensitivity of this array to small deviations in any of its design parameters. For example, changing element spacing from 0.25 wavelength (**fig. 12**) to 0.272 results in the formation of two additional minor lobes at

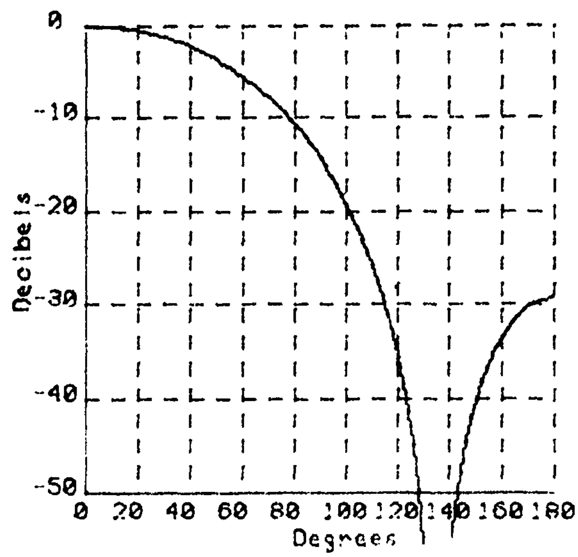

fig. 14. Four-element, 4-square configuration, 1/8-wave spacing −135 degree and −270 degree phasing, 1:1:1:1 current ratios.

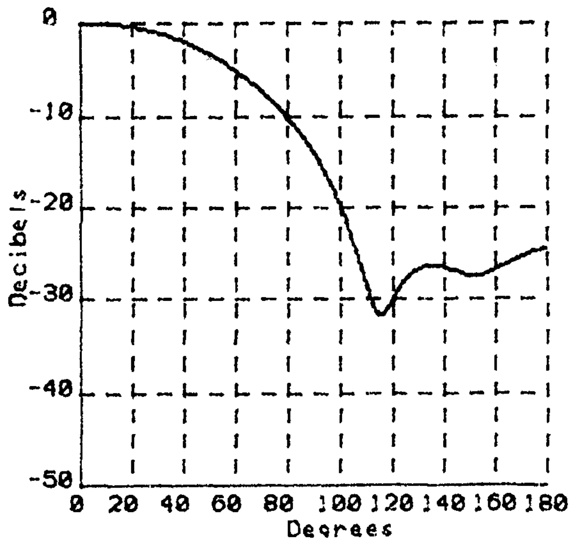

fig. 15. Four-element, 4-square configuration, 1/4-wave spacing −90 degree and −190 degree phasing, 1:1:1:1 current ratios.

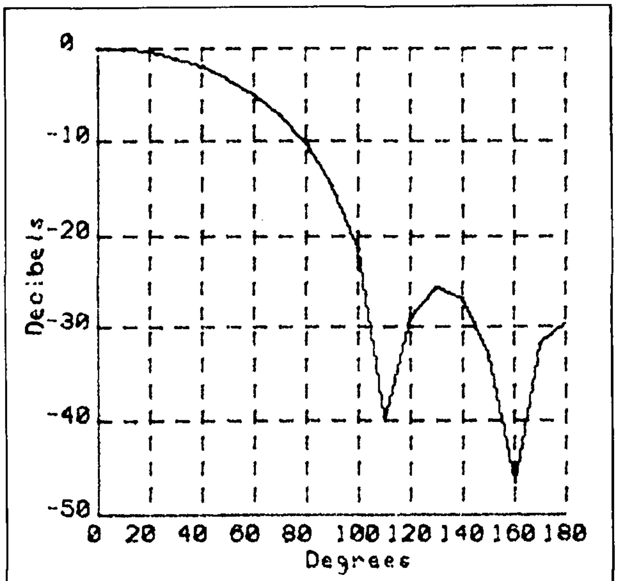

fig. 16. Four-element, 4-square configuration, 0.272-wave spacing −90 degree and −180 degree phasing, 1:1:1:1 current ratios. Poor granularity (plotted in steps which are too large).

130 and 230 degrees (**fig. 13**). If the phasing is changed from exact multiples to −90 and −190 degrees (**fig. 15**), the additional lobes have formed but without definition to the nulls. The same sensitivity is shown to small dissimilarities in drive current ratios among the elements.

array of arrays

This name refers to an antenna arranged to consist of arrays which are themselves arrayed. The simplest example consists of two two-element arrays configured as a square. Two adjacent elements are treated as reference elements fed in phase, and the remaining two elements are also fed in phase but displaced −90 degrees. Current amplitudes are all equal. This scheme allows switching the main lobe in four directions, except these are offset 45 degrees from the diagonal directions. In combination with the 4 square feed network we could have eight directions. As commented in connection with the triangular array, since the half-power beamwidth of the 4-square fed connection is about 95 degrees, switching in a different feed network for these additional directions appears to be a needless complication. (Perhaps more useful would be the provision for a separate feed network for optimum F/B operation of the array in the 80-meter phone and CW subbands.)

With 1/4 wave spacing, −90 degrees phasing and all elements fed equal current (amplitude), the half-power beamwidth is about 140 degrees as seen in **fig. 7**. The −25 dB or better F/B width is a paltry 40 degrees, although at 180 degrees azimuth it approaches infinity. Except for increased gain over a single two-element array, the performance of this antenna is not notable and is not nearly equivalent to that obtained from the same physical layout when connected as a 4-square.

As noted earlier for simpler arrays, the 1/8-wavelength-spaced 4-square field pattern is nearly identical to the 1/4-wavelength-spaced pattern. In each type of array examined we can note differences which show improvements in all characteristics over its equivalent larger size array. However, closer spacing means high mutual coupling, which in turn means even greater sensitivity to element variations. Such an array is difficult to provide identical field patterns for all switchable directions. Unless you have prior experience with these arrays and have equipment for accurate measurements of the self- and mutual impedances of the array elements, you are strongly advised to avoid these closely spaced arrays.

One-eighth-wavelength arrays present an interesting challenge and some opportunities. They offer the same array in much less area — and two-band operation is possible. But if the height of the elements is 1/8 wavelength at the lowest frequency, the self-im-

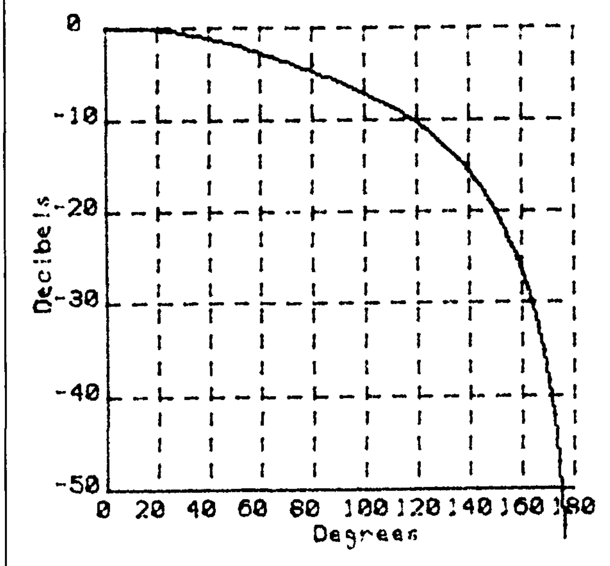

fig. 17. Four-element, twin two-element array, arranged in a square configuration, 1/4-wave spacing −90 and −90 degree phasing, 1:1:1:1 current ratios. This array of arrays does *not* use the 4-square feed network.

pedances are going to be quite low. The resistive component will be about 6 ohms — not easy to work with, and placing a high premium on the need for a low-loss groundplane. If the element length is significantly more than 1/4 wavelength at the highest frequency, temporary sectioning of these elements has to be provided so that impedance measurements can be made (the elements have to be electrically separable into 1/4 wavelength or shorter sections for the measurements).

other possibilities

An interesting possibility is a five-element array which places an additional element in the center of a 4-square. Since this element is always occupying the same electrical position in the array regardless of beam direction it represents no increased switching complication. F/B ratio width can be increased over that of the 4-square even if −30 dB is used as the limiting criterion. Alternatively, if a way could be found to keep the "outrigger" elements from entering into the act, this arrangement of elements could also be operated as crossed three-element in-line arrays. Although there would be some loss in gain, the tradeoff is a significant improvement in F/B depth.

conclusion

So much for the theoretical design. With the concepts and suggestions reviewed in this article, I hope I have given experimenters the tools and some ideas for selecting and siting an array.

The next part of this series deals with self- and mutual impedances; how to measure them, and, most crucial of all, what to do with them. Until we know the driving-point impedances, the feed network design cannot proceed.

acknowledgement

I am indebted to WB6SXV who derived the field strength algorithm used.

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2. Robert Booth, WB6SXV, "General Solution for Phased Array Antenna System for Vertical Monopoles," unpublished.
3. James L. Lawson, W2PV, "Simple Arrays of Vertical Antenna Elements," *QST*, May, 1971, page 22.
4. Dana Atchley, Jr., W1CF, Harold E. Stinehelfer, ex-W2ZRS, Joseph F. White, Phd., "360°-Steerable Vertical Arrays," *QST*, April, 1976, page 27.
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ham radio

vertical phased arrays: part 3

Array impedances, measurements, and calculations

This is the third in a series of articles on phased vertical arrays by K2BT.

In Part 1 (May), the author examined essential design parameters, and more importantly, the *assumptions* underlying design. (In the past, incorrect assumptions have misled designers into constructing less than optimally performing arrays.)

Part 2 (June) continued with relative power plots of two- to four-element arrays indicating the correlation between physical and electrical (phase) spacing and performance.

This month, K2BT's discussion includes the determination of self- and mutual-impedances, the importance of an extensive ground system, and a tabulation of mutual and driving point impedance values for some popular vertical phased arrays. — **Editor**.

In Part 2[1] various types of arrays were examined and relative power (in dB) plots were shown. We saw how specific physical arrangements of elements, current amplitude ratios, and phase displacements formed beams. By varying current amplitude ratios and phases, the forward beam width or the rejection characteristics of a given physical array were modified. The question now is how can these drive conditions be created in a real array? To do this we need information about element impedances in order to design the feed network.

Knowledge of self-impedance and mutual impedances, as well as factors that influence them, is essential because everything will be either directly or indirectly affected by these parameters.

self-impedance

The self-impedance of an antenna at any frequency is a function of the element length, its radius, ground plane loss, and coupling with other nearby antennas. Strictly speaking, the last two items are not components of self-impedance. However, when measuring self-impedance, both may be present in the reading of *apparent* self-impedance.

Although resonant elements are not required for an array, their use simplifies calculations and provides the following advantages:

1. An *open-circuited* 1/4-wavelength element presents virtually no coupling. This simplifies measurement procedure and ensures best conditions for accuracy of self- and mutual impedance readings.

2. The resistive component of self-impedance is normally higher than ground loss resistance which results in reasonable efficiency.

3. Ground plane evaluations and comparisons are easier to make because more information is available about the 1/4-wavelength resonant antenna than about other types of vertical antennas.

element length and radius

An article on Yagi design by James Lawson, W2PV,[2] provides data on the relationship between an element's resonant length and its radius. (When using this source, be sure to refer to error corrections.[3]) It's important to use a full wavelength when calculating length-to-radius ratio, K, for W2PV's equations. For determining parameters of a resonant grounded 1/4-wavelength element, I have revised W2PV's chart as shown in **fig. 1**. In the Yagi antenna

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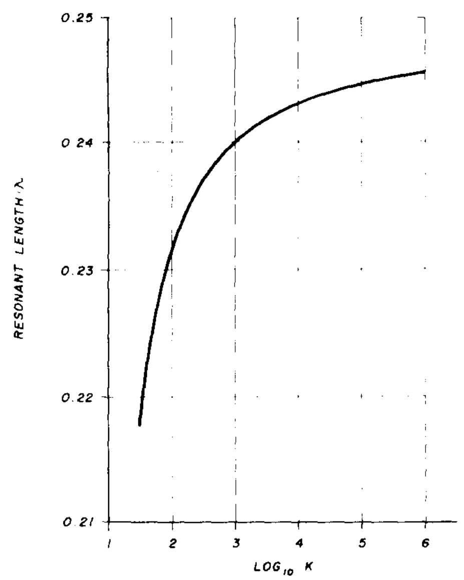


fig. 1. Resonant length of a quarter-wavelength grounded element as a function of k (operating wavelength to element radius ratio).

design, emphasis was placed on the reactance component of self-impedance, ignoring the effect that radius has upon the resistive component. In an all-elements-driven array as compared to a parasitic array, it is more important to know this effect. A review of the Amateur literature yields a range of values for a 1/4-wavelength vertical resistive component of impedance; these values are probably all correct. Any disparity is probably due to the different antenna element diameters that are used. The theoretical self-impedance of a physical 1/4-wavelength high vertical is 36.5 + j21[4] which assumes the use of an infinitely conducting ground plane and an infinitely thin element. Obviously neither of these conditions is physically realizable. However, even if an infinitely thin element could be used, it still would have to be shortened to achieve resonance — and in so doing the resistive component would decrease. A real element, having real thickness, would reduce resistance some more since it requires a further reduction in length in order to achieve resonance. Kraus[5] shows that l/r ratios in the range of 60 to 1000 are equal to a resistance variation from 34 to 36 ohms, with 35 ohms as an average value. He uses an element's actual length when calculating l/r. The comparable data for *reactance* change compiled by W2PV would show a variation for K from 240 to 4000. When resistance is plotted against the logarithm of K, we see a virtually straight line, showing a slow reduction in resistance as the element diameter is varied from 1.5 to 24 inches.

ground planes

Considerable controversy surrounds the subject of required ground plane size and its influence on antenna performance. The ground plane essentially establishes an image antenna to represent the other half of a dipole. The better that image, the lower the ground loss and the lower the radiation angle. How large the ground plane should be is answered by examining the near field (within the first 1/2 wavelength), and far field (to at least 6 wavelengths) components. The near field requirements for proper pattern formation is satisfied by a ground system composed of wire radials; a sufficient quantity allows us to get quite close to the theoretical resistance. At the lower frequencies the far field usually must be left to nature, since it would be prohibitively expensive to provide so large a radial wire or mesh ground system. Even the large a-m broadcast antennas are located in salt marshes whenever available to take advantage of the high conductivity of earth for many wavelengths beyond the reach of the radials.

My experience correlates closely with the work reported by Jerry Sevick, W2FMI.[6,7] His graph of resistance versus number of radials used on 40 meters is applicable for 80 meters as well. I used radials averaging 0.3 wavelength in length, composd of PVC No. 24 hookup wire, and laid them on the ground. The only difference noted is that resistance did not decrease as rapidly as his graph shows. For instance, I never found resistance below 40 ohms with 40 radials, but at 60 radials and greater the data correlated more closely. This discrepancy is probably attributable to the differences in soil conductivity; the land under my array is part of a moraine, and consequently represents very low conductivity earth. All indications are that with *120* 1/4-wavelength radials, resistance of a resonant 1/4-wavelength vertical is within a half ohm of the theoretical value regardless of the underlying soil conductivity. Another effect I noticed which W2FMI did not comment upon was that as radials were added, the element length had to be slightly but continually adjusted upward to maintain resonance.

coupling with other antennas

The attempt to approach the theoretical self-impedance value can be frustrated by inadvertent coupling of the antenna under test to another antenna. As will be seen when discussing mutual impedance, the effects are subtle and can be easily mistaken for ground plane differences. These effects can go in both directions — you may think you are achieving theoretical self-impedance with a 30-radial ground plane, or conversely that a 120-radial ground plane has several ohms loss. If you encounter either of these indications, suspect coupling with another antenna (or something acting like one even if you don't "see" it). Another indication of this problem is a significant departure (at 80 meters — several inches) in element length for resonance. I had a tower guy wire (adequately broken up with insulators, I thought) whose lowest section ran to an anchor at the base of a tree. This section was approximately 1/4 wavelength and it found sufficient ground conductivity in the tree roots to present lossy coupling to one of my array elements. Though I knew that element wasn't right, I could not see anything that would act as a resonant antenna around it. That guy wire didn't look as if it had a ground plane! The solution was to insulate it at the anchor, thus decoupling the section of guy wire.

I am sure many Amateurs will identify with this frustrating experience: the first element of a multi-element array is erected and adjusted for resonance. The length is carefully recorded and the second erected. Then, letting the first element remain connected to its feed cable, the second element is checked for resonance, found too long, and is readjusted downward. Reconnecting the second element to its feeder, the first element is now found too long. And so it continues; the result is that the elements end up considerably shortened below their uncoupled resonant length. This is mutual coupling at work and the error was in failing to open-circuit other elements when making self-impedance measurements. Other elements, at or near resonance and within about 0.35 wavelength of the antenna being measured, will manifest inductive coupling. Unless you're aware of what is happening, you may diagnose this inductive reactance to be due to the element's being too long. Shortening it will bring it to "resonance" and this may be accompanied by a satisfactory reduction in resistance (perhaps even below theoretical), but all this changes when the second element is open-circuited. It is well to remember that this situation can also occur inadvertently with a conductor not recognized as acting as an antenna. However, as we shall soon see, this same effect — mutual coupling — is the very same process used to advantage to create field enhancement and cancellation in arrays.

mutual impedance

Coupling between elements is a function of element lengths, distance between elements, relative attitudes of elements (e.g., parallel, co-linear, echelon), and ground plane losses. Ground losses are not actually a component of theoretical mutual impedance but in a practical situation they become a part of the *apparent* mutual impedance. (Mutual impedance is a term that relates to the interaction of two or more antennas which are close enough to each other to cause their driving impedances to be different from their self-impedances.) The unit of measurement — ohms — may be, like any impedance, resistive or reactive, or both. Such antennas are coupled by an impedance which appears to be in common with all elements. (Driving point impedance calculations only require the mutual impedance between pairs — that is, two elements at a time be measured.) Mutual impedance between antennas is similar to mutual inductance between coupled coils; the impedance relationship can be both depicted and its value measured in the same way. In **fig. 2** the driving point impedance Z_1 or Z_2 of each vertical as measured at either set of terminals reacts to the presence of the other vertical as though its self-impedance Z_{11} or Z_{22} had a common impedance Z_{12} in series with it. Z_{12} is, by definition:

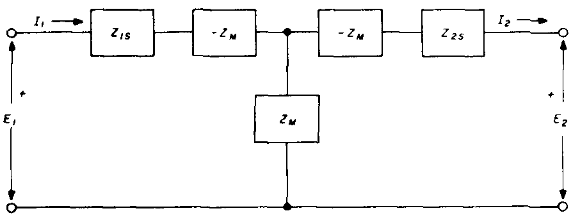


fig.2. Four-terminal equivalent network for two verticals.

$$Z_{12} = -E_2/I_1$$

Although useful mathematically, it doesn't provide a practical basis for measurement. The voltage and current relationships existing in a system of antenna elements, each mutually coupled to one another, have the same form as the voltage and current in a general network. Writing their mesh equations produces:

$$E_1 = I_1Z_{11} + I_1Z_{12} + \dots + I_nZ_{1n}$$

$$E_2 = I_1Z_{21} + I_2Z_{22} + \dots + I_nZ_{2n}$$

$$\vdots$$

$$E_n = I_1Z_{n1} + I_2Z_{n2} + \dots + I_nZ_{nn}$$

where $E_1, E_2 \dots, E_n$ are voltages applied to elements 1,2,...N

$I_1, I_2 \dots, I_n$ are element drive currents

$Z_{11}, Z_{22} \dots, Z_{nn}$ are element self-impedances

$Z_{12}, Z_{21} \ldots, Z_{1n}, Z_{2n}$ are mutual impedances and are denoted by dual subscripts which are always different. As in general networks, mutual impedances with the same subscripts but with reversed positions, (e.g., Z_{12} and Z_{21}), describe the identical impedance (from the Reciprocity Theorem).

If the equation for each drive voltage is divided by that element's drive current, the following driving point impedance terms are obtained:

$$Z_1 = E_1/I_1 = Z_{11} + I_2 Z_{12}/I_1 + \ldots + I_n Z_{1n}/I_1 \quad \textbf{(1)}$$

$$Z_n = E_n/I_n = I_1 Z_{n1}/I_n + I_2 Z_{n2}/I_n + \ldots + Z_{nn}$$

Notice that each element's driving point impedance consists of its self-impedance and includes terms for the mutual impedances between it and each of the other elements. The *influence* of the mutual impedances upon the driving point impedance is a function of the drive currents (amplitude and phase) to other elements. Although at first glance these equations appear quite formidable and look like there are too many unknowns for solution, this is not the case. Having selected an array configuration and the driving current ratios and displacements for the field plot, we already know what the currents need to be.[1] If we could find a way to reduce the complexity and consequently the number of unknowns, a means for deriving mutual impedances might be devised. Fortunately there is one. Since each mutual impedance we need to know exists between only two elements, we can write a simpler set of equations:

$$E_1 = I_1 Z_{11} + I_2 Z_{12}$$

$$E_2 = I_1 Z_{12} + I_2 Z_{22}$$

If the terminal of element 2 is connected to its ground plane, the drive voltage E_2 becomes zero and:

$$\begin{aligned} E_1 &= I_1 Z_{11} + I_2 Z_{12} \\ O &= I_1 Z_{12} + I_2 Z_{22} \end{aligned} \quad \textbf{(2)}$$

Solving for the driving point impedance yields:

$$Z_1 = E_1/I_1 = Z_{11} - (Z_{12})^2/Z_{22}$$

and solving for the mutual impedance Z_{12} gives

$$Z_{12} = \pm \sqrt{Z_{22}}\,(Z_{11} - Z_1) \quad \textbf{(3)}$$

Note that all references to voltages and currents have been eliminated. We are now in a position to find all the remaining unknowns.

mutual impedance measurement

Provided the elements are 1/4 wavelength or less, the procedure is: open-circuit all elements; measure the self-impedance of element 1; connect element 2 terminal to its ground plane; measure the driving point impedance of element 1; and open-circuit element 2.

If there are additional elements, connect element 3 terminal to its ground plane; measure the driving point impedance of element 1; and open-circuit element 3.

Following the same sequence, all remaining elements are measured from element 1. When completed, a similar set of measurements are taken from element 2, starting with self-impedance and then measuring the various pairs of driving point impedances, and so on with each remaining element. This procedure allows each element to be individually treated as the reference element of each pair of elements for mutual impedance measurements. When completed, the same mutual impedance will have been read from each side of every pair. This provides a check on previously determined calculations. I am continually amazed (even though I know it is supposed to happen) by the close coincidence of the resulting value for mutual impedance as determined from either element of a pair! This occurs, as it should, even when the two self-impedances are quite *different*.

using 1/2-wavelength elements

What if the elements are significantly longer than 1/4 wavelength, specifically a 1/2 wavelength? Open-circuiting these elements from the ground plane will not decouple them (in all likelihood, coupling will be found to increase if the length is exactly a 1/2-wavelength). Means for temporarily sectioning other elements into two electrically separate halves must be provided so that self-impedances are measured with the temporary sectioning reconnected and *that element* connected to its ground plane. I have no experience with this situation but I believe the array can be driven properly, provided the high impedance at the bases of the elements can be handled.

In antenna texts, mutuals are always referred to current loops (maximum current points). Mutuals derived from measurements as above are referred to the base of the elements. These are quite different values, just as self-impedances differ according to whether they are measured at a voltage or current loop.

mutual impedance calculations

Data is taken from a 40-meter 4-square array with elements spaced 0.272 wavelength at 7.0 MHz. The elements are not alike, not resonant, and the ground plane is quite lossy. Data are shown for two elements and mutual coupling was measured from each.

table 1. List of mutual resistance and reactance between two physical 1/4-wavelength verticals separated by 0 through 1.5 wavelength spacings.

spacing	R	X	spacing	R	X
0	+36.57	+21.27	.80	−9.25	+6.13
.05	+35.83	+12.14	.85	−6.66	+8.15
.10	+33.67	+3.77	.90	−3.75	+9.28
.15	+30.22	−3.55	.95	−.78	+9.50
.20	+25.70	−9.59	1.00	+2.00	+8.87
.25	+20.40	−14.18	1.05	+4.38	+7.52
.30	+14.63	−17.22	1.10	+6.16	+5.61
.35	+8.75	−18.71	1.15	+7.26	+3.36
.40	+3.11	−18.72	1.20	+7.63	+0.97
.45	−1.99	−17.39	1.25	+7.28	−1.33
.50	−6.27	−14.97	1.30	+6.30	−3.35
.55	−9.53	−11.71	1.35	+4.81	−4.92
.60	−11.66	−7.94	1.40	+2.99	−5.94
.65	−12.61	−3.97	1.45	+1.00	−6.35
.70	−12.43	−0.13	1.50	−.94	−6.15
.75	−11.25	+3.32			

Equation 3 is used to calculate the mutual impedance.

Measurements from Element A (referenced as Element #1)

Element A	$Z_{11} = 45.73 + j\,8.19$	Self-impedance of A
Element B	$Z_{22} = 42.53 + j\,5.72$	Self-impedance of B
Element A	$Z_1 = 46.98 + j15.66$	Driving point impedance of A with B grounded
	$Z_{12} = 12.53 - j12.95$	Calculated mutual impedance

Measurements from Element B (referenced as Element #1)

Element B	$Z_{11} = 42.53 + j\,5.72$	Self-impedance of B
Element A	$Z_{22} = 45.73 + j\,8.19$	Self-impedance of A
Element B	$Z_1 = 44.20 + j12.79$	Driving point impedance of B with A grounded
	$Z_{12} = 12.63 - j13.34$	Calculated mutual impedance

Note the following:

1. There is a nomenclature interchange for the self-impedances of the elements, denoting the change in reference element for the measurement of mutual coupling.

2. There is only a small increase in resistive component when measuring the effect of coupling, requiring a highly accurate impedance bridge.[8]

3. At this spacing, the effect of coupling is decidedly inductive on the measured element.

4. There is reasonably good correspondence in the mutual impedance calculation from either side of the pair of elements, despite the differences in the individual elements.

5. The measured mutual impedance is quite different from theoretical values. (See **table 2.**)

As a further verification of measurements and calculations, this test is useful and instructive: With element 2 connected to its ground plane, drive element 1 from a 50 to 100 watt source while measuring current at the terminals of each element. The ratio of the current flowing in element 2 to element 1 is equal to the ratio of the mutual impedance to element 2 self-impedance:

$$I_2/I_1 = -Z_{12}/Z_{22}$$

(This identity is a rearrangement of **eq. 2.**)

Since ratios are involved, the only restraint on the current measuring device is that it be linear. Although phase angles are difficult to measure when the reference points are located at some distance, current amplitudes can be measured and this identity is useful as a verification of impedance measurements and calculations, even if only the magnitude of the mutual impedance vector can be obtained. When performing this test, if there are more elements, open circuit them. If driving with more than 50 watts be careful of those open-circuited elements; don't provide a ground return through your body. You may be surprised to find how much energy is being coupled.

The calculations for mutual impedances require a square root extraction. Which sign to use? As general guidance, the polar vector angle of the root is *always* lagging except at spacings less than about 0.15 wavelengths. For a specific calculation the pattern of sign changes seen in published sources is an aid. Mutual resistance and reactance vary with element separation in the nature of a damped sine wave, starting with both signs positive at zero separation and proceeding through cyclic sign variations thereafter. For example, suppose at 1/4-wavelength separation with 1/4-wavelength elements your calculator or computer produces the square root extraction $-13.7 + j15.1$ (polar notation $20.4 \quad +132.2°$). The polar angle shows lead and it should be lagging. Looking at published sources we see confirmation for this. Subtracting 180° from the polar vector angle will produce the correct signs for resistance and reactance. To aid in determining signs I have converted the table of mutual resistances and reactances shown by W2PV, to grounded physical 1/4 wavelength values in **table 1**.

The question arises: "Why bother measuring mutual impedances? Why not use published values from antenna texts?" The best answer is another question: "Why not also use textbook values for self-impedance?" Most Amateurs measure self-impedance because they want to be sure the element length is resonant at the frequency of interest or because they know from experience that the actual self-impedance can differ considerably from the theoretical value. Theoretical mutual impedance derivations are quite complex and solutions often use different simplifying assumptions. The result is that few textbook sources — except those which obtained data from a common origin — agree exactly. Regardless of source, the following assumptions apply: infinitely conducting ground plane; infinitely thin element; and element lengths measured in physical wavelengths. Element radius has a relatively small effect on mutuals. The element length assumption can be determined from the values for zero separation between elements (see first line in **table 1**). This is the self-impedance of a single element and may be recognized as identical with theoretical self-impedance. (Applies to equal length element data only.) For example, the value 36.5 + j21 means that physical 1/4-wavelength elements had been assumed. The length difference (over resonant length) will not seriously affect driving point impedance calculations, but the assumption of lossless self-impedances will. **Table 2** lists mutual impedance between 1/4 wavelength high elements from several sources compared to an average of 16 measurements I have made.

table 2. Values of mutual impedance between two quarter-wavelength high verticals. Data from five different sources. (Gehrke's entry represents *measured* data for a real vertical over a real ground.)

source	mutual impedance (0.272 spacing)	mutual impedance (0.385 spacing)
Brown	17.49 −j17.01	2.96 −j18.47
Jasik	17.47 −j16.01	6.00 −j17.50
Jordan	17.55 −j16.37	1.66 −j18.99
Mushiake	17.51 −j15.70	4.80 −j18.75
Gehrke	13.20 −j16.24	0.20 −j16.61

The resistive component differs most. Despite these differences, if no means of measurement is *available, there is something to be said for using theoretical values;* at least there is recognition they exist rather than ignoring them entirely. However, as I have previously emphasized, the significance of deviation from optimum drive conditions increases with the complexity of the array. When I first became aware of the need to take mutual impedances into account for the feed network, I used theoretical values. There was improvement in F/B, but it was still far from what is achievable.

You may have wondered if an element driving-point impedance could have a negative resistive component, and if so, what that means. This is entirely possible with arrays of more than two elements, particularly with close spaced arrays or arrays employing non-unity current ratios. Elements exhibiting this condition are being driven by energy coupled from other elements; instead of *receiving* any drive from its feeder, this element is *sending* drive back into the feed network. This is still a coupled passive system, *in equilibrium*, merely observing the law of conservation of energy.

calculations of driving-point impedances

Using **equation 1**, I have calculated and listed in **table 3** the driving-point impedances of several arrays discussed in **Part 2** using measured mutuals. (For smaller spacings, values were estimated based on extrapolations of my data). For a comparison, the 4-square array driven impedances are also calculated using mutual impedances from **table 1**.

Data common to all calculations:

Element effective radius = 0.7 inch
Element height = 62.7 feet
Self-impedance = 36.4 + j0 ohms
Frequency = 3.8 MHz

notes and comments

1. The 3 element in-line and the 1/8-wavelength 4-square have elements which exhibit substantial negative resistance components in their driving point impedances.

2. Nearly all driving point impedances show substantial reactance, requiring some care in establishing correct phasing.

3. All arrays except one exhibit unlike driving impedances, *ruling out equal power* distribution networks where *equal current* amplitude is intended.

4. Note the difference in driving point impedances in

table 3. Mutual and driving point impedance values for some popular vertical phased arrays.

array	current ratio	mutual impedances	driving point impedances
2-element, $\lambda/4$ spacing*	1/1; 0°, −90°	$Z_{12} = 15 - j15$	$Z_1 = 21.4 - j15$ $Z_2 = 51.4 + j15$
3-element in-line, $\lambda/4$ spacing	1/2/1; 0°, −90°, −180°	$Z_{12} = Z_{23} = 15 - j15$ $Z_{13} = -9 - j13$	$Z_1 = -6.6 - j21$ $Z_2 = 51.4 + j0$ $Z_3 = 79.4 - j39$
2-element, $\lambda/2$ spacing	1/1; 0°, −180°	$Z_{12} = -9 - j13$	$Z_1 = 45.4 + j13$ $Z_2 = 45.4 + j13$
triangular array, 0.289λ spacing	1/0.5/0.5; 0°, −90°, −90°	$Z_{12} = Z_{23} = Z_{13}$ $= 10 - j16$	$Z_1 = 28.4 - j10$ $Z_2 = 78.4 + j4$ $Z_3 = 78.4 + j4$
4-square array, $\lambda/4$ spacing	1/1/1/1; 0°, −90°, −90°, −180°	$Z_{12} = Z_{13} = Z_{24} = Z_{34}$ $= 15 - j15;$ $Z_{14} = Z_{23} = 3 - j17.5$	$Z_1 = 3.4 - j12.5$ $Z_2 = 39.4 - j17.5$ $Z_3 = 39.4 - j17.5$ $Z_4 = 63.4 + j47.5$
4-square array, $\lambda/4$ spacing (using table 1 mutual impedance data)	1/1/1/1; 0°, −90°, −90°, −180°	$Z_{12} = Z_{13} = Z_{24} = Z_{34}$ $= 20.4 - j14.18;$ $Z_{14} = Z_{23} = 8.41 - j18.72$	$Z_1 = -0.37 - j22.08$ $Z_2 = 44.81 - j18.72$ $Z_3 = 44.81 - j18.72$ $Z_4 = 56.35 + j59.52$
2 × 2 array of arrays, $\lambda/4$ spacing	1/1/1/1; 0°, 0°, −90°, −90°	$Z_{12} = Z_{13} = Z_{24} = Z_{34}$ $= 15 - j15,$ $Z_{14} = Z_{23} = 3 - j17.5$	$Z_1 = 18.9 - j33$ $Z_2 = 18.9 - j33$ $Z_3 = 83.9 + j3$ $Z_4 = 83.9 + j3$
4-square array, $\lambda/8$ spacing	1/1/1/1; 0°, −135°, −135°, −270°	$Z_{12} = Z_{13} = Z_{24} = Z_{34} =$ $= 30 - j3,$ $Z_{14} = Z_{23} = 25 - j9$	$Z_1 = -1.27 - j13.18$ $Z_2 = 18.97 - j4.76$ $Z_3 = 18.97 - j4.76$ $Z_4 = -10.78 + j21.67$

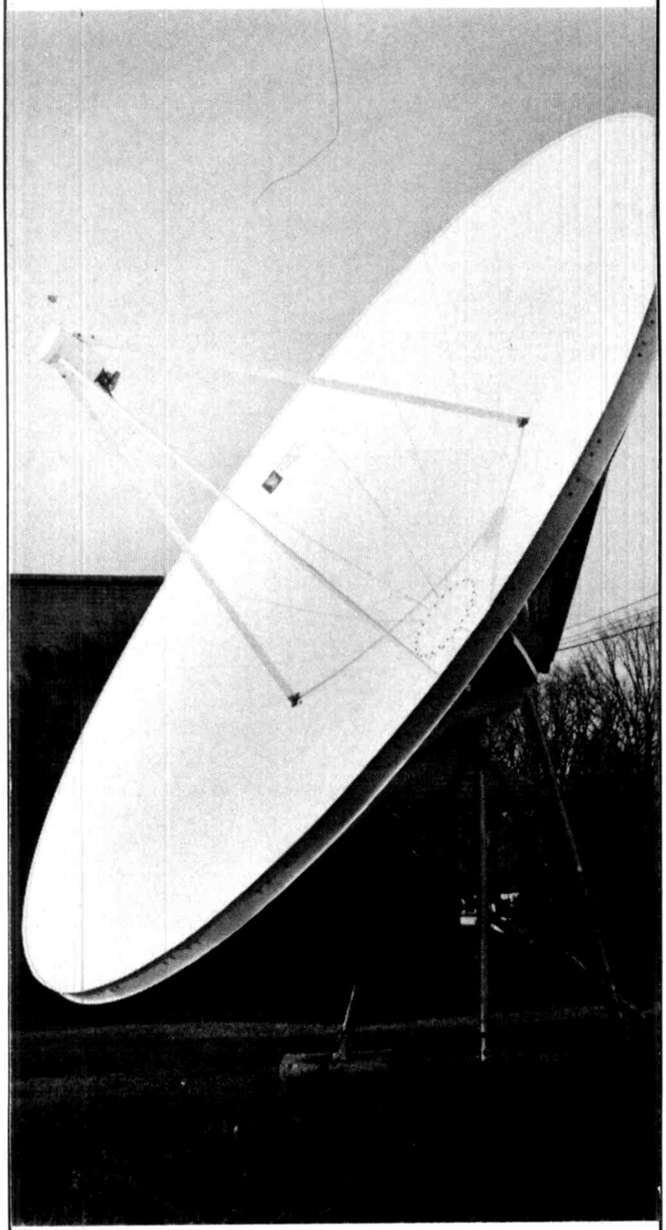
*This 2-element, 1/4-wavelength spaced array is probably the most common phased array configuration used by Amateurs today. Please note that the driving point impedances are different.
Editor.

the 1/4 wavelength spaced 4-square using actual mutual impedances as compared to the use of theoretical values. Current and phases in the latter case will not occur as intended in a real array.

5. Note the 2 element 1/2 wavelength spaced array (not shown in Part 2). Because of the equal driving impedances, here is one of the few instances of an array which operates as intended regardless of feeder length, as long as they are equal and a 1/2 wavelength delay line is inserted in series with one of them. Except for VSWR, Z_0 of coax is not important. The antenna pattern in this case is *not* a function of the coaxial cables Z_0 (characteristic impedance) though the VSWR still is.

We tend to become accustomed to thinking of an antenna, just as any discrete component, as having a fixed impedance at any frequency. The concept that elements within an array present impedances that are determined by other element drive currents (amplitude and phase) is, at first, difficult to appreciate. That these impedances may have negative components of resistance also can be a bit unsettling. Yet when an array is looked at mathematically as a general network which includes the impedance branches represented by mutual impedances, these seemingly unusual effects can be seen to be physical realities. Consequently, the rest of this coupled system, the feed network, must be designed for these driving impedances as the terminations.

If we expect to switch directions with this array, then we need to be sure that each physical element presents the same driving point impedance appropriate to the electrical position in the array it is assuming. I have found that equalizing self-impedances is the best means for doing this. Each element is adjusted for length to present the identical reactance (if resonance is the objective, then this is zero reactance). Assuming all elements have the same radius, radials are added to those elements showing higher resistive components. At the 100 radial level, it is not unusual for a spread of +20 radials to occur among the ground planes of the elements in this effort at equalization.



summary

We have worked our way through the design of vertical phased array antennas. A number of typical arrays were examined, as well as the current requirements of each element and the driving point impedances that must exist to cause the array to operate as designed. What remains is to design the feed network which will create conditions as they must appear, not at the element terminals, but at the end of the feed lines coming from those terminals. By now you are aware, if you weren't already, that feed lines are an integral part of the feed network.

There is no unique network which achieves the necessary current amplitude ratios and phase displacements. We can get to that objective in a number of different ways. In the next article the design task will be of use A,B,C,D parameters in single matrices as a tool. If this technique is new to you, I believe you will find this approach most interesting. You will see that this is a powerful and versatile means of network design, useful not just for antenna arrays, but for other network applications as well.

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In commenting on vertical phased arrays, several writers have cautioned against placing arrays near trees. The apparent assumption is that trees represent resonant loss elements or somehow disturb the field so that the radiated pattern will be changed. I remain unconvinced. At wavelengths 40 meters and longer, I have measured self- and mutual impedances of elements, among trees, at all seasons of the year without seeing any significant changes that are not also seen on a pair of 40-meter elements completely away from trees. Small variations (0.3 to 0.5 ohms) are seen in self-impedances, depending upon soil moisture conditions, which are reflected in mutual impedance measurements. Since all elements are affected in the same way, these small changes cannot significantly affect radiation patterns. Examination of published mutual impedance data indicates that the presence of conductive elements, *resonant or not*, within about 0.1 wavelength of an element will significantly affect mutual impedance in unanticipated ways. Prudence would therefore dictate that nothing conductive, or even partially so, which could act as an antenna be allowed within that distance. If despite this precaution array patterns are indeed disturbed, my advice is to look for something that may be acting as a real conductive antenna in the immediate area of the array, or to re-evaluate the feed network. — K2BT

ham radio

vertical phased arrays: part 4

Feed network design using L-match circuits, π and tee coax-equivalent circuits

Previous articles of this series on vertical phased arrays[1,2,3] concentrated on the design of the physical aspects of arrays: element length, radius, spacing and ground planes. The latest article[3] dealt with electrical measurements of the arrays and calculation of driving-point impedances. Knowing the required drive current amplitude and phase for each element of the array pattern selected, and knowing the measured values of self- and mutual impedances, we can calculate the driving-point impedance of these elements. The importance of this cannot be over-emphasized; because of mutual impedance effects between elements, driving-point impedances of elements in an array are not fixed entities. Each element's driving-point impedance depends upon the amplitude and phase of the drive currents — not just upon its own drive current, but upon the amplitude and phase of the drive current of every element in this array. A change of the current amplitude or phase to any element results in a driving-point impedance change of *every* element together with a change in *all* current amplitudes and phases. Examination of the set of simultaneous equations defining these driving-point impedances illustrates this relationship:

$$Z_1 = E_1/I_1 = Z_{11} + I_2Z_{12}/I_1 + I_3Z_{13}/I_1 + \cdots + I_n/Z_{1n}/I_1 \qquad (1)$$

$$Z_2 = E_2/I_2 = I_1Z_{12}/I_2 + Z_{22} + I_3Z_{23}/I_2 + \cdots + I_nZ_{2n}/I_2$$

$$Z_n = E_n/I_n = I_1Z_{1n}/I_n + I_2Z_{2n}/I_n + I_3Z_{3n}/I_n + \cdots + Z_{nn}$$

where $Z_1, Z_2, \cdots, Z_n$ are element driving-point impedances

$E_1, E_2, \cdots, E_n$ are element impressed voltages

$I_1, I_2, \cdots, I_n$ are element drive currents

$Z_{11}, Z_{22}, \cdots, Z_{nn}$ are element self impedances

$Z_{12}, Z_{13}, Z_{23}, \cdots, Z_{1n}, Z_{2n}, Z_{3n}$ are mutual impedances between pairs of elements

(All terms can be complex.)

For example, suppose the drive current I_2 to element 2 changes. Since I_2 appears in the equation for every element, all driving-point impedances, currents and voltages are affected. The self and mutual impedances do *not* change, even though it is the mutual impedances that cause this interaction.

An array is a coupled system, automatically adjusting to any change with a new set of currents and phases, which again simultaneously satisfies all the equations. There is an infinite number of such solutions, but *only a few* result in useful array patterns. A feed network must be designed that when connected to the terminals of each element, applies the proper voltage amplitude and phase, causing the required drive currents to flow. If this condition is met, then all conditions are met. (It may now be clearer why I have been emphasizing the importance of physical and electrical symmetry of the elements.) As the array direction is switched, each port of the feed network continues to "see" the same driving-point impedance it was designed for, even though each port is now feeding a different element. Exact symmetry is probably the most difficult condition to meet in practice because it depends upon more than just simple duplication of physical elements; it also depends on duplication of the environment adjacent to each element: for example, ground planes or other nearby conductors that might act as antennas.

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feed networks

Just as there are an infinite number of solutions to the set of equations defining the driving-point impedances of an array, there are almost as many ways to design networks fitting the one solution required. Some designs are better than others, resulting in more bandwidth for usable F/B (front-to-back) ratio or low SWR. As a general rule, the simpler (that is, the fewer stages in the network), the better, but there are exceptions.

For superior F/B performance, for all network designs, the designer must know the driving-point impedance of each element. In this respect, vertical phased arrays are more critically affected by element variations than Yagis are by height variations. For multiple-element arrays, cookbook recipe duplication attempts are almost guaranteed to miss optimum current drive conditions by 10 percent. *This is enough to reduce the F/B performance by 50 percent* or more.

basic design objectives

Some basic design decisions must first be made: what type of circuit elements should be used? Should the design have the objective of a 1:1 SWR match to the array feedline? Feed networks may be devised using coaxial cables as circuit elements. Although simple in construction, there may be technical and cost drawbacks. Series or stub coaxial cable sections, provided one has a wide enough selection of different characteristic impedances and lengths, could be used for a network that matches to the feedline. The cost would be high and bandwidth narrower than the array's intrinsic F/B capability.

As a special case, for arrays operating with 90-degree current phase multiples and equal current amplitude ratio element pairs,[3] an approach suggested by W7EL[4] makes use of the unique characteristic of a 1/4-wavelength line to produce a constant 90-degree phase displacement between input voltage and output current, independent of the load termination. If two such lines are connected to a common feedpoint, equal current will flow into the loads regardless of their termination impedances. The current phase displacement between the two loads is 0 degrees, but this may be changed to 180 degrees by insertion of a 1/2-wavelength line in series with one of the lines. (The phase displacement of a 1/2-wavelength line also is independent of the load impedance.) A 90-degree current phase displacement, as was pointed out earlier in this series,[1] cannot be obtained by insertion of a 1/4-wavelength line when the termination is reactive. Therefore, instead of inserting an additional 1/4-wavelength line, a lumped-constant phase correction circuit based upon the calculated driving-point impedances is inserted. It provides a drive current phase of 90 degrees and the correct amplitude at the element(s). The input voltage amplitude and phase to the correction circuit must be designed to be the same as that of the common connection point of the array. SWR of the array can be minimized by proper choice of the characteristic impedance of the coaxial cable feeder lines but cannot be designed for a 1:1 condition. This approach, based upon this unique characteristic of 1/4-wavelength lines, which are also the element feedlines, is limited to arrays where this length is able to physically reach the directional switch. If not possible, a further 1/2-wavelength line has to be added to each feedline to maintain the basis of the design concept.

This article provides sufficient information to enable the reader to design a feed network for any conceivable array. There are no restrictions on array spacings, current amplitude ratios or phase displacements. The elements must be alike, but they need not be resonant. Conventional or not, if the array you've been able to fit onto your property has a useful pattern, a feed network can be designed to drive it at the needed conditions. Such versatility requires complete design freedom — freedom to use any characteristic impedance, to transform to any input resistance, regardless of reactive load impedances. Coax is an excellent means for transmission of RF energy between physically separated points. If it also fills a role as a specific circuit element, so much the better. But as a circuit element where the physical spacing does not require it, coax is confining; with only two characteristic impedance choices commonly available (there are perhaps two more, but neither is easy to find), one is constantly making compromises and designing around this limitation. Furthermore, ease of circuit adjustment is not notable. On the other hand, π and tee coax-equivalent lumped-constant circuits may be designed for any exact characteristic impedance or any electrical length, whether lagging or leading phase, and are easily adjusted. And surprisingly, low impedance lumped constant circuits of the same levels as coaxial transmission lines display comparable characteristics, even when designed for fairly large single increments of phase displacement. **Table 1** compares coaxial cable with a 45-degree π circuit cascaded with a 45-degree tee circuit operated as a 1/4-wavelength transformer. Off-frequency phase variation and development of input reactance compares very favorably with coaxial transmission line.

Rounding out the list of network building blocks are the shunt and series input L-match transforming circuits. Included are the two special cases of this circuit, where the series or shunt branch is absent, which I will call a Parallel and a Series impedance circuit, respectively. **Figs. 1A** through **1F** are schematics of all of these circuits.

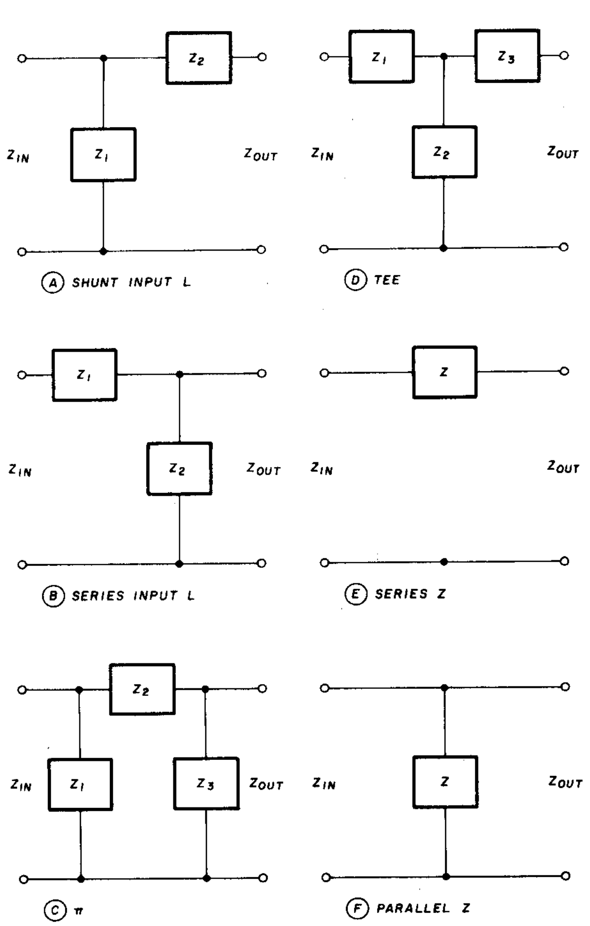

fig. 1. Basic network "building blocks."

why a 1:1 SWR?

For multi-element arrays the objective of a 1:1 match to the array feedline does not stem from an obsession with SWR. *A low SWR provides no significant measure of an array's efficiency or usable F/B bandwidth.* Designing for an SWR of 1:1 simplifies network design calculations and electrical tests. However, the real value is the instant array condition conveyed every time an SWR measurement is made. A failing relay in a directional switch or a network malfunction is quickly detected, even if this circuit is to an element requiring very little power. Such a failure may raise the SWR from 1:1 to 1.1:1, for example, while the same failure in an array normally showing 2:1 will not be noticed. At "smoke" test time it removes uncertainty; a 1:1 SWR represents an unambiguous confirmation of the accuracy of array measurements, network design, and construction.

As an illustration of the step-by-step design procedure for an array feed network, I will use the popular 2-element array. The same array will be used to show the error that arises in many 2-element vertical array feed arrangements. From Part 3 of this series,[3] the driving-point impedances of two 1/4-wavelength resonant elements spaced 1/4-wavelength apart with unity current ratio and 90 degree phase displacement, are:

Element 1	**Element 2**
$Z_1 = 21.4 - j15$	$Z_2 = 51.4 + j15$
$I_1 = 1 \angle 0^\circ$	$I_2 = 1 \angle -90^\circ$

Assuming 50 ohm 1/4-wavelength feedlines, using a Smith Chart or by calculation, the element driving-point impedances rotated to the input ends of the feeders are:

$$Z_1 = 78.3 + j54.91 \qquad Z_2 = 44.82 - j13.08$$

At element drive conditions of 1 ampere with a phase displacement of −90 degrees between elements, the voltages and currents that must be applied to the inputs of these feeders in polar form are:

$$I_1 = 0.52 \angle 55.0^\circ \qquad I_2 = 1.07 \angle 16.3^\circ$$
$$E_1 = 50 \angle 90^\circ \qquad E_2 = 50 \angle 0^\circ$$

Notice that the current phase change in the two equal 1/4-wavelength feeders are 55 degrees and 106.3 degrees (90 degrees + 16.3 degrees). Next a 1/4-wavelength 50-ohm delay line is added to the feedline from element 2. Rotating the impedance to the end of the delay line we find these conditions:

$$Z_2 = 51.4 + j15$$
$$I_2 = 1 \angle 90^\circ$$
$$E_2 = 53.54 \angle 106.3^\circ$$

These are the conditions that must exist at the input ends of the feeders from each element for the assumed drive conditions. The current phase delay through the delay line is less than 90 degrees, (90 degrees − 16.3 degrees, or 73.7 degrees, the difference between the input and output angle). Observe that the input voltage amplitudes and phases are not alike at the input ends of the coaxial lines from the two elements. But these two terminals are normally connected together; clearly two different voltages can't coexist here. Since the difference is fairly small, the actual drive conditions that result if connected anyway will be acceptable, though the F/B ratio will diminish. The choice of 1/4-wavelength element feeders just happened to provide this fair agreement. I estimate the actual phase displacement between elements to be about 115 degrees and the current amplitude ratio about 1.15. The 1/4-wavelength delay line didn't produce a 90-degree delay and the delays in the two equal length feeder lines were unequal; these are all quite different results from what is often assumed to occur.

Some writers have assured us we can use any length feeders as long as lengths are kept equal. Let's see how we fare following this advice using 3/8-wavelength 50-ohm feeders for the same array:

Element 1	**Element 2**
$Z_1 = 21.4 - j15$	$Z_2 = 51.4 + j15$
$I_1 = 1 \angle 0°$	$I_2 = 1 \angle -90°$
$E_1 = 26.13 \angle -35.03°$	$E_2 = 53.54 \angle -73.73°$
135° Feeder	**135° Feeder**
$Z_1 = 63.58 - j53.98$	$Z_2 = 37.43 + j2.67$
$I_1 = 0.58 \angle 148.56°$	$I_2 = 1.17 \angle 51.66°$
$E_1 = 48.39 \angle 108.22°$	$E_2 = 43.97 \angle 55.75°$
	1/4 wavelength delay line
	$Z_2 = 66.46 - j4.75$
	$I_2 = 0.88 \angle 145.75°$
	$E_2 = 58.60 \angle 141.66°$

Note: all impedances, voltages, and currents are input conditions — that is, looking towards the load.

Using 3/8-wavelength feeders, the input voltages required to be applied to each chain are very different. If these terminals are tied together the drive conditions to the elements will be far from intended. Conclusion: element feeders are an integral part of the feed network for a phased array; their circuit characteristics must be taken into account.

designing for optimum drive

While it is possible to solve for the undesirable drive conditions that would result from making this connection, why bother? It is better to start with the correct design. While doing so, suppose a 1:1 SWR match to a 50-ohm array feedline is included. This would require that the paralleled input impedances of the networks from each element be 50 ohms pure resistance. Assuming lossless conditions, we can go back to the resistive components of the element driving-point impedances for this determination. These are 21.4 and 51.4 ohms, respectively. At 1 ampere to each element the total drive power is the sum of the I^2R inputs, or 72.8 watts. Using the relationship $E^2/R = W$, and substituting 50 ohms (the characteristic impedance of the array feed-line) for R:

$$E^2 = 50(72.8), \text{ or } E = \sqrt{3640} = 60.33 \text{ volts}$$

Having established the array feedline voltage amplitude for this drive power, we can calculate the required resistive inputs for each element's network. Rearranging, $R = E^2/W$:

$$R_1 = 3640/21.4 = 170.09 \text{ ohms for network, No. 1 input}$$

$$R_2 = 3640/51.4 = 70.82 \text{ ohms for network, No. 2 input}$$

As a check on calculations it is useful to do the parallel conductance calculation:

$$1/R_1 + 1/R_2 + \cdots + 1/R_n = 1/Z_0 \qquad (2)$$

Starting again at the end of the 3/8-wavelength feeder to element 1, one possibility is an L-match network, transforming directly to 170.09 ohms pure resistance. L-match circuit component calculations involve a square root extraction, guaranteeing at least two solutions. (Under certain circumstances, there may be four solutions.) While all solutions produce the intended transformation, they do so with differing phase displacements, with at least one of those displacements being a leading phase. Remembering that element 2 is starting 90 degrees behind the first, fewer stages in the network usually result if a leading phase L-match is chosen for element 1.

Shown in **table 2**, beginning with the driving-point impedances and working forward to the common

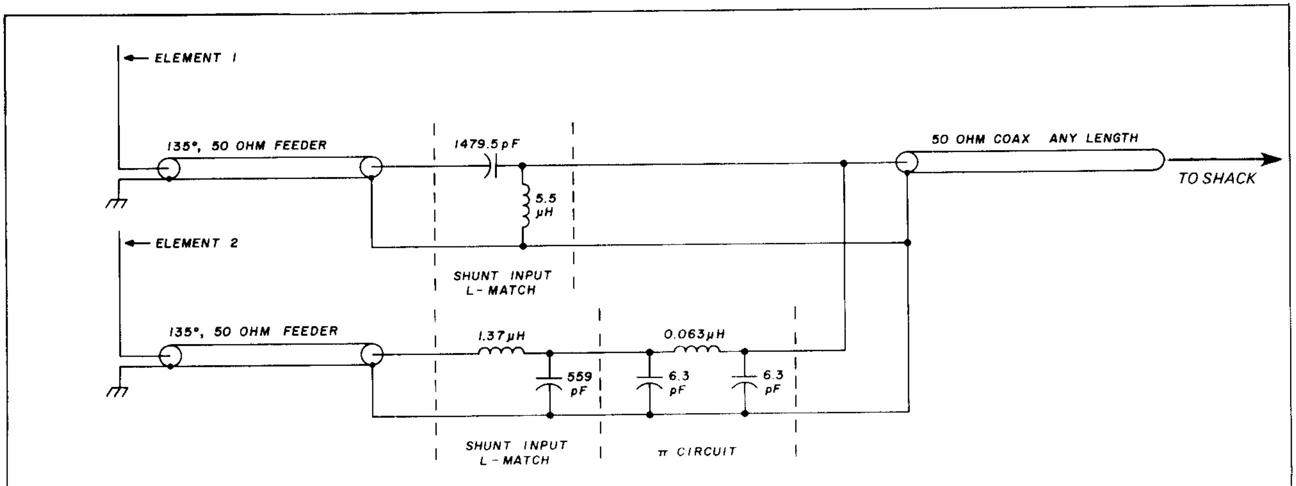

fig. 2. Matching network for a 2-element array using 3/8 λ coaxial feeders at a 3.8 MHz design frequency.

connection of the array, are the input parameters of each circuit. **Fig. 2** shows the schematic and component values at the design frequency of 3.8 MHz.

At the inputs of each network chain, E_1 and E_2 are equal in amplitude and phase; the two inputs may be connected together without disturbing drive conditions. Their paralleled resistive inputs represent a 50-ohm resistive load, as designed. The π coaxial-equivalent network was added to the element 2 chain only to show how this type of network circuit is used to match the voltage phase at the common connection of the array network. In this example the agreement that happened to be achieved at the input to the 2 element shunt L-match is sufficient; the π network can be omitted.

4-terminal networks. The design procedure for producing exact matching at the required array conditions has been demonstrated. Before proceeding with other examples, the design equations for these circuits are presented. All network circuit components are reactances and assumed to be lossless. Subscript **a** denotes the series load termination components, $R_a + jX_a$, instead of the more commonly used $R_L + jX_L$, to avoid any confusion with jX_L, as an inductive reactance.

L-match circuit. This circuit can take two forms (see **fig. 1A** and **1B**), termed *Shunt Input* and *Series Input* L-matches. Though this circuit consists of only two components, its analysis is relatively complex. The calculations for either form include a square root extraction, resulting in two possible sets of components for any desired impedance transformation. Either set works; one set often has a leading phase angle, while the other may lag. The absolute value of the angles are not necessarily equal nor always of opposite sign! The component set may have the same reactance sign; that is, both may be inductive (+) or both capacitive (−). The circuit is sometimes referred to as an L-C match, but it could also be an L-L or C-C match. A more apt description is L-match, taken from the similarity of its schematic representation to the letter "L".

Shunt input L-match. The series arm component X_2, must be calculated first, since its value is used in the calculation for the second component:

$$X_2 = -X_a \pm \sqrt{R_a (R_{in} - R_a)} \; ohms \qquad (3)$$

$$X_1 = -\left[\frac{R_a^2 + (X_2 + X_a)^2}{X_2 + X_a}\right] ohms \qquad (4)$$

where R_a and X_a are the series equivalents of the load termination and
R_{in} is the desired input (pure) resistance.

Close attention must be paid to signs. A positive result indicates an inductance, while a negative sign is a capacitance.

Series input L-match. X_2 must be calculated first. Note that X_2 is the shunt arm of this circuit, however.

$$X_2 = \frac{-R_{in}X_a \pm \sqrt{R_{in}R_a(R_a^2 + X_a^2 - R_{in} R_a)}}{R_{in} - R_a} \text{ ohms} \qquad (5)$$

$$X_1 = \frac{-X_2[R_a^2 + X_a(X_2 + X_a)]}{R_a^2 + (X_2 + X_a)^2} \text{ ohms} \qquad (6)$$

Which form should be used? Usually, the shunt input L-match is the only form possible if R_{in} is equal to or greater than R_a. Besides, the arithmetic is easier! The series input L-match is used when R_{in} is less than R_a. There is a set of circumstances, however, in which the series form can be used even if R_{in} is greater than R_a. Inspection of the equation for the series form calculation of X_2 will show this case when R_{in} is greater than R_a and when $(R_a^2 + X_a^2 - R_{in} R_a)$ is equal to or greater than zero. Four solutions, two series and two shunt L-matches, then result. These additional options, if available, are often useful, allowing a smaller phase displacement or more (physically) realizable set of components as a result.

π coaxial-equivalent circuit (fig. 1C). The π circuit and the shunt input L-match will be found to be the most frequently used circuits for vertical phased array feed networks. The type used here is set up as a reversible network — that is, the input and output can be interchanged without affecting operation, just as with coaxial cable. Reactances X_1 and X_3 are always equal and if capacitive, then X_2 is inductive. At the design frequency this particular configuration shows the same properties as coax. As the frequency is varied, the phase displacement starts differing from that obtained with coax, the difference being larger the greater the equivalent "length" of the circuit. If, instead, multiple sections, each an equal increment of the total phase displacement, are cascaded, the combined network approaches coaxial cable characteristics. This should be expected since the equivalent circuit of coax is a series of infinitesimally small π sections. The design equations are relatively simple:

$$X_2 = Z_0 \sin\theta \; ohms \qquad (7)$$

$$X_1 = X_3 = -\frac{Z_0 \sin\theta}{1 - \cos\theta} \; ohms \qquad (8)$$

where Z_0 is the required characteristic impedance
θ is the electrical length in degrees

A positive sign indicates an inductance while a negative sign indicates capacitance.

If a *leading* phase, say 30 degrees, is desired, this would be equivalent to 330 degrees in electrical length if coaxial cable were used. Substituting 330 degrees in these equations causes X_2 to be negative and X_1 and X_3 to be positive; the appropriate capacitance and inductances can then be calculated from the relations:

$$C = 1/\omega X \text{ and } L = X/\omega \tag{9}$$

where $\omega = 2\pi f$, f = *frequency in Hz*

Half-wave section (180 degree electrical length) π circuits are taboo, since the calculated circuit values are physically unrealizable. At the least, two separate 90-degree sections are suggested to achieve this "electrical length."

Tee coaxial-equivalent circuit (fig. 1D). This circuit is used in the same applications as the π circuit. Alternated with π networks in equal increments of electrical length, network characteristics can be made to equal or exceed coax (assuming coax of the same characteristic impedance is available for comparison). For applications requiring a leading phase displacement only one inductance (for the shunt arm) is necessary, sometimes simplifying construction. The design equations are:

$$X_2 = -Z_0/\sin\theta \text{ ohms} \tag{10}$$

$$X_1 = X_3 = \frac{Z_0(1 - \cos\theta)}{\sin\theta} \text{ ohms} \tag{11}$$

where Z_0 = required characteristic impedance
θ = electrical length required in degrees

As with the π network, a positive sign indicates inductive reactance and a negative sign, capacitive reactance. Also, 180 degree sections cannot be physically realized and require at least a 2-section cascaded network to achieve that displacement.

Series impedance circuit (fig. 1E). This circuit is used when R_{in} is equal to R_a and the load has a reactance X_a. The series matching impedance is simply the reactance of the opposite sign.

$$X = -X_a \text{ ohms} \tag{12}$$

Parallel impedance circuits (fig. 1F).

$$X = -\left[X_a + \frac{R_a^2}{X_a}\right] \text{ ohms} \tag{13}$$

The parallel matching reactance has the opposite sign of the parallel equivalent reactance of the load. The series and parallel circuits can be thought of as a shunt input L-match — with one of its circuit branches either equal to infinity or zero impedance, respectively.
These conditions occur when $R_{in} = R_a$ or

$$X_a = \sqrt{R_a(R_{in} - R_a)}$$

Either circuit should be considered, particularly when the load has a relatively large reactance compared to its resistive component. The circuit is simple, and cascaded with a following L-match circuit, results in a broader bandwidth network.

design limitation and other considerstions

Some design hints may be helpful to understanding the use of these circuits:

1. The L-match circuits first require selection of the input resistance wanted, transforming from any output impedance. Phase displacement, however, cannot be pre-defined, though the direction, lead or lag, may be chosen.

2. Single L-match impedance transformation ratios exceeding 5-to-1 should be avoided. Above that ratio, expect to see increased frequency sensitivity and resultant reduction in bandwidth. For high ratios, consider transforming in step increments of resistance using several L-matches or combinations of L-match and π or tee circuits (the latter as 1/4-wavelength transformers).

3. In this particular application, π and tee circuits are always designed for pure resistance terminations. These circuits are designed to act as a 1/4-wavelength transformer, or as a specific coaxial-equivalent length, leading or lagging, of transmission line. Choose any characteristic impedance, but keep in mind that large (more than ± 90 degree) increments of angular displacement, especially at high impedances, reduce bandwidth.

4. Cascaded circuits may each have a capacitor at their common connection points, which are then in parallel. For example, see **fig. 3** showing a 567 pF and 392 pF capacitor at a common connection point. The two values may be added and a single capacitor placed at that junction. However, until the network has been tested, it is useful to keep the circuits independent for separate adjustment.

designing networks for multi-element arrays

Armed with the design equations for simple 4-terminal networks, we can now examine feed networks for arrays consisting of several elements. If the array is one requiring a phase angle multiple, for example, 0, 90, and 180 degrees, or 0, 100 and 200 degrees, and all feedlines are equal in length, the simplest network may result if the middle element is treated as if it were the reference element of the array. The respective networks for the array end elements are designed to lead and to lag the middle element. Then neither has to be designed to span a large angular displacement, and fewer stages result.

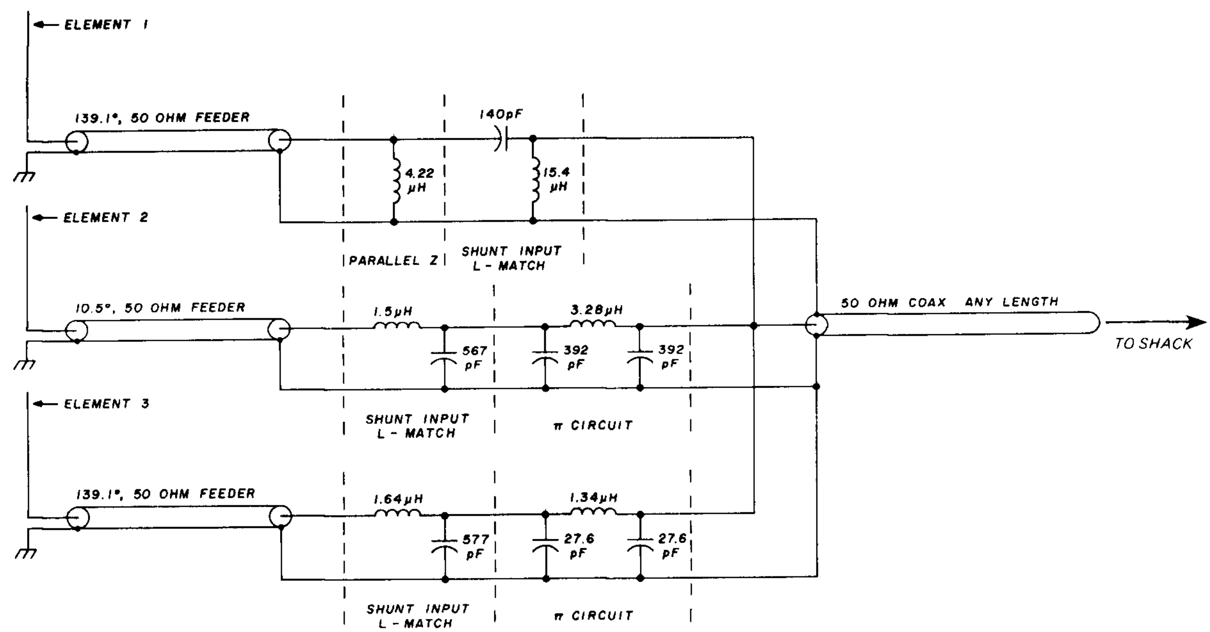

fig. 3. Matching network for a 3-element in-line array at a 3.8 MHz design frequency.

3-element in-line array. This array has a particularly deep F/B ratio extending over a wide azimuthal sector. We should be especially interested in taking advantage of this capability. Since the middle element has the same drive-point impedance regardless of array direction, there is no need to make its feeder equal in length to other feeders. Assuming the directional switch is located five feet from the middle element, equal length end element feeders are brought to the center area. At 3.8 MHz, using 0.66 velocity factor coax, these are 66 feet (139.1 degrees) and for the center element, 5 feet (10.5 degrees) with a Z_0 of 50 ohms. Assuming an array of 3 resonant 1/4-wavelength elements, spaced a quarter-wavelength apart, with current amplitude ratios of 1,2,1 and phase relationships of 0, −90, and −180 degrees, respectively, the driving point impedances are $Z_1 = 15.4 - j17$, $Z_2 = 36.2 + j0$ and $Z_3 = 75.4 + j43$. (Part 3 showed these values incorrectly).[5] As was done with the 2-element array example, the feed network is matched to the 50-ohm array feedline. The sum of the I^2R input power terms, assuming 1 ampere to the first and third elements and 2 amperes to the middle element, is 235.6 watts. Using the $E^2/R = W$ relationship, this establishes an amplitude of 108.54 volts at the array feedline connection. At that point the input impedances for each element's network are the pure resistances:

$$Z_1 = 764.94 + j0$$
$$Z_2 = 81.25 + j0$$
$$Z_3 = 156.23 + j0$$

The sequence of input parameters at each junction of the networks is shown in **table 3**.

The resulting network is shown in **fig. 3**. Illustrated in this example is the application of the parallel circuit and the use of leading and lagging phase L-match circuits. Here, element 1 is used as the reference element of the feed network. A parallel impedance circuit is used to transform the impedance seen at the input end of the feeder to a pure resistance. This is then transformed to the pure resistance required for the chain with a shunt input L-match chosen to produce a leading phase change. The resulting input voltage then becomes the objective for the other two network chains.

Triangular array. The triangular array feed network demonstrates still another technique for simplifying a feed network. Since elements 2 and 3 operate at identical conditions, the inputs of their transmission line feeders may be paralleled and fed from a common network. **Fig. 4** shows the two feeders connected to a shunt input L-match, and being transformed directly to a resistive input. This is then cascaded with a tee circuit having a sufficiently leading phase displacement to equal the voltage amplitude and phase of the element 1 network. The array termination is designed to match a 50-ohm transmission line. Part 3 incorrectly showed the driving-point impedance of element 1. The correct impedance is $Z_1 = 20.4 - j10$. **Table 4** shows the sequence of input parameters at each network junction.

4-square array. The 4-square array obviously requires a more complicated drive network. The look-

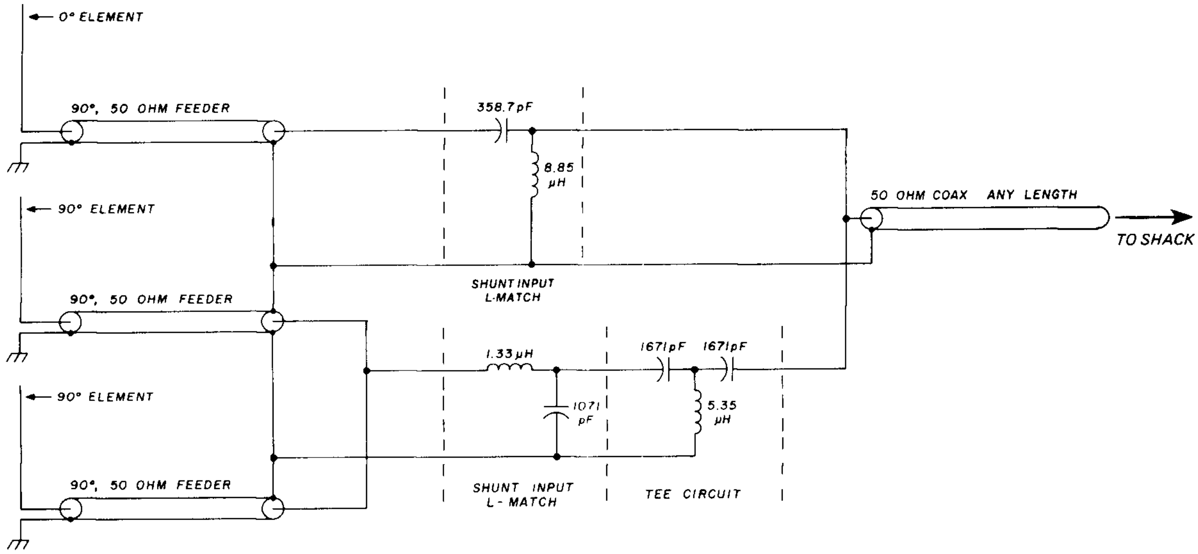

fig. 4. Matching network for a triangular array at a 3.8 MHz design frequency.

alike middle elements present the opportunity to connect their feedlines in parallel, simplifying the design somewhat. The 4-square element driving-point impedances are highly reactive, making any drive network more frequency dependent. There is the further question of the directions the driving-point impedances take as frequency is changed from design center. The relatively small amount of measurements I have taken to examine this question indicate a not unexpected similarity to Yagis. Array performance falls apart more rapidly on the low-frequency side of design center than on the high side. Whether a drive

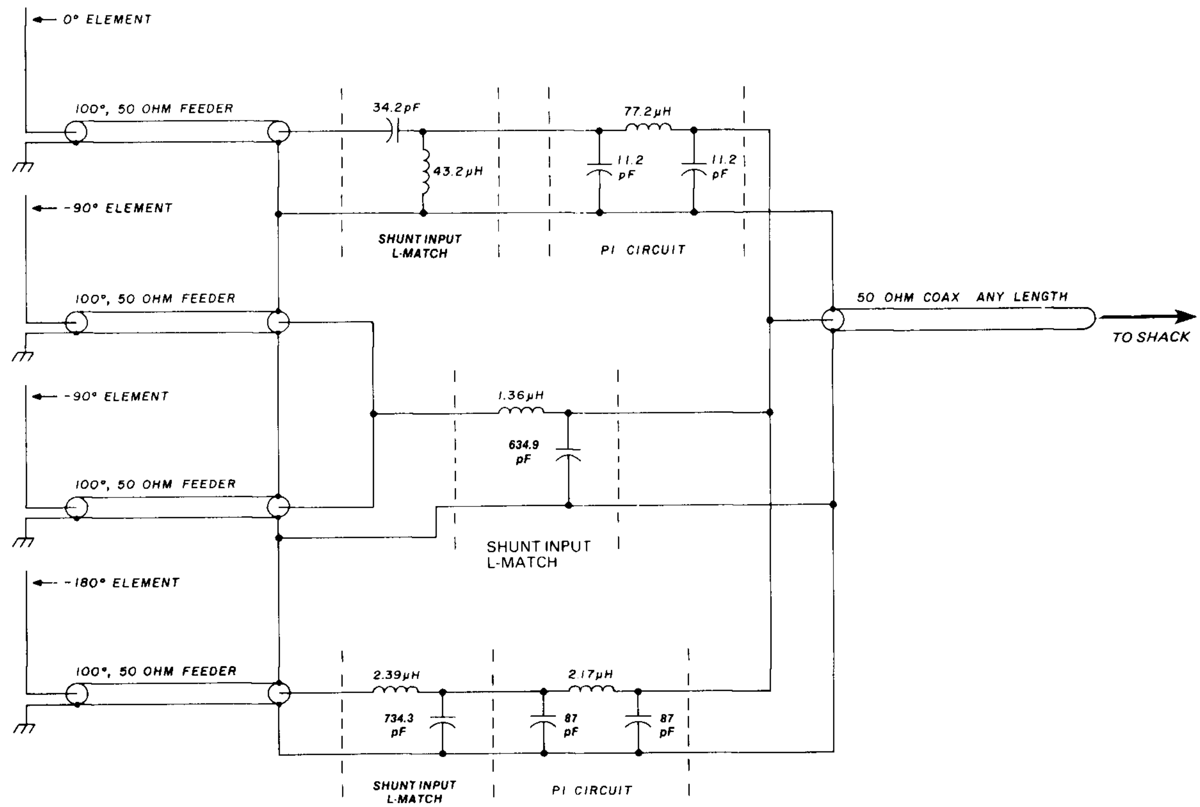

fig. 5. Matching network for a 4-square array at a 3.8 design frequency.

table 1. Comparison of impedance, voltage and current phase, and SWR variations with frequency, of 90 degree length of coax and a cascaded 45 degree π circuit and 45 degree tee circuit, both acting as a 50-ohm characteristic impedance 1/4-wavelength transformer. Load termination is 75 ohms pure resistance; the center design frequency is 3.75 MHz.

frequency MHz		coax	π and tee
3.55	Z_{in}	33.46 − j2.33	33.45 − j1.86
	I_o/I_i	0.67 $\angle -86.8^\circ$	0.67 $\angle -86.33^\circ$
	E_o/E_i	1.49 $\angle -82.82^\circ$	1.50 $\angle -83.15^\circ$
	SWR	1.5	1.498
3.65	Z_{in}	33.37 − j1.16	33.36 − j0.91
	I_o/I_i	0.67 $\angle -88.4^\circ$	0.67 $\angle -88.15^\circ$
	E_o/E_i	1.5 $\angle -86.4^\circ$	1.5 $\angle -86.58^\circ$
	SWR	1.5	1.5
3.75	Z_{in}	33.33 + j0	33.33 + j0
	I_o/I_i	0.67 $\angle -90^\circ$	0.67 $\angle -90^\circ$
	E_o/E_i	1.5 $\angle -90^\circ$	1.5 $\angle -90^\circ$
	SWR	1.5	1.5
3.85	Z_{in}	33.37 + j1.16	33.36 + j0.88
	I_o/I_i	0.67 $\angle -91.6^\circ$	0.67 $\angle -91.88^\circ$
	E_oE_i	1.5 $\angle -93.6^\circ$	1.5 $\angle -93.39^\circ$
	SWR	1.5	1.5
3.95	Z_{in}	33.46 + j2.33	33.45 + j1.72
	I_o/I_i	0.67 $\angle -93.2^\circ$	0.67 $\angle -93.79^\circ$
	E_o/E_i	1.49 $\angle -97.18^\circ$	1.5 $\angle -96.74^\circ$
	SWR	1.5	1.498

Note: I_i, E_i equal input current and voltage, respectively. I_o, E_o equal output current and voltage, respectively.

network can be designed to reduce this tendency is a question. Perhaps the best alternative is to set the design center frequency on the low end of the intended operating range, recognizing that the optimum F/B bandwidth is a narrow frequency band of about 2 to 3 percent.

Using the driving-point impedances from Part 3 for a 4-square consisting of 1/4-wavelength resonant elements, spaced a quarter-wavelength apart, and phased 0, −90, −90, −180 degrees with current amplitude ratios 1,1,1,1, respectively, input parameter sequences of a suggested drive network are

table 5. Network input parameters for a *4-square array*.

	element 1	element 2	element 3	element 4
	Z_1 3.4 − j12	Z_2 39.4 − j17.5	Z_3 39.4 − j17.5	Z_4 63.4 + j47.5
	I_1 1 $\angle 0^\circ$	I_2 1 $\angle -90^\circ$	I_3 1 $\angle -90^\circ$	I_4 1 $\angle -180^\circ$
	E_1 12.47 $\angle -74.18^\circ$	E_2 43.11 $\angle -113.95^\circ$	E_3 43.11 $\angle -113.95^\circ$	E_4 79.22 $\angle -143.16^\circ$
100° coax	Z_1 403.97 + j387.1	Z_2 62.39 + j22.57	Z_3 62.39 + j22.57	Z_4 22.73 − j11.37
	I_1 0.09 $\angle 46.88^\circ$	I_2 0.79 $\angle -12.43^\circ$	I_3 0.79 $\angle -12.43^\circ$	I_4 1.67 $\angle -74.97^\circ$
	E_1 51.33 $\angle 90.66^\circ$	E_2 52.73 $\angle 7.46^\circ$	E_3 52.73 $\angle 7.46^\circ$	E_4 42.44 $\angle -74.97^\circ$
		elements 2 & 3 paralleled		
L-match	Z_1 2141 + j0	Z_2, Z_3 92.39 + j0		Z_4 114.83 + j0
	I_1 0.04 $\angle -17.37^\circ$	I_2, I_3 0.92 $\angle 42.04^\circ$		I_4 0.74 $\angle 15.2^\circ$
	E_1 85.32 $\angle -17.37^\circ$	E_2, E_3 85.32 $\angle 42.04^\circ$		E_4 85.32 $\angle 15.2^\circ$
π circuit	Z_1 2141 + j0			Z_4 114.83 + j0
	I_1 0.04 $\angle 42.04^\circ$			I_4 0.74 $\angle 42.04^\circ$
	E_1 85.32 $\angle 42.04^\circ$			E_4 85.32 $\angle 42.04^\circ$

table 2. Network input parameters for a *2-element array*.

element 1	**element 2**
Z_1 $21.4 - j15$	Z_2 $51.4 + j15$
I_1 $1\angle 0^\circ$	I_2 $1\angle -90^\circ$
E_1 $26.13\angle -35.03^\circ$	E_2 $53.54\angle -73.73^\circ$
135° coax feeder	**135° coax feeder**
Z_1 $63.58 - j53.98$	Z_2 $37.43 + j2.67$
I_1 $0.58\angle 148.56^\circ$	I_2 $1.77\angle 51.66^\circ$
E_1 $48.39\angle 108.22^\circ$	E_2 $43.97\angle 55.75^\circ$
shunt input L-match	**shunt input L-match**
Z_1 $170.09 + j0$	Z_2 $70.82 + j0$
I_1 $0.36\angle 96.25^\circ$	I_2 $0.85\angle 95.03^\circ$
E_1 $60.33\angle 96.25^\circ$	E_2 $60.33\angle 95.03^\circ$
	π circuit
	Z_2 $70.82 + j0$
	I_2 $0.85\angle 96.25^\circ$
	E_2 $60.33\angle 96.25^\circ$

table 3. Network input parameters for a *3-element in-line array*.

element 1	**element 2**	**element 3**
Z_1 $15.4 - j17$	Z_2 $36.2 + j0$	Z_3 $75.4 + j43$
I_1 $1\angle 0^\circ$	I_2 $2\angle -90^\circ$	I_3 $1\angle -180^\circ$
E_1 $22.94\angle -47.83^\circ$	E_2 $72.4\angle -90^\circ$	E_3 $86.80\angle -150.3^\circ$
139.1° Coax	**10.5° Coax**	**139.1° Coax**
Z_1 $47.42 - j67.6$	Z_2 $36.71 + j4.35$	Z_3 $27.77 + j20.68$
I_1 $0.57\angle 159.27^\circ$	I_2 $1.98 + \angle -82.33^\circ$	I_3 $1.65\angle -36.83^\circ$
E_1 $47.06\angle 104.32^\circ$	E_2 $73.49\angle -75.59^\circ$	E_3 $56.98\angle -0.25^\circ$
parallel Z	**shunt input L**	**shunt input L**
Z_1 $143.19 + j0$	Z_2 $81.35 + j0$	Z_3 $156.23 + j0$
I_1 $0.33\angle 104.32^\circ$	I_2 $1.33\angle -34.59^\circ$	I_3 $0.69\angle 28.24^\circ$
E_1 $47.06\angle 104.32^\circ$	E_2 $108.54\angle -34.59^\circ$	E_3 $108.54\angle 28.24^\circ$
shunt input L	**π circuit**	**π circuit**
Z_1 $764.94 + j0$	Z_2 $81.35 - j0$	Z_3 $156.23 + j0$
I_1 $0.14\angle 40.01^\circ$	I_2 $1.33\angle 40.01^\circ$	I_3 $0.69\angle 40.01^\circ$
E_1 $108.54\angle 40.01^\circ$	E_2 $108.54\angle 40.01^\circ$	E_3 $108.54\angle 40.01^\circ$

given in **table 5** with the schematics shown in **fig. 5**.

Reaching the center area of a 4-Square array whose elements are spaced 1/4 wavelength with 1/4 wavelength feeders is a problem. Coax with a velocity factor greater than 0.71 is required. Unfortunately, most foam dielectric cables have velocity factors around 0.71, allowing little or no slack for placement of and connections to a directional switch and the feed network. Since a lumped-constant network imposes no restriction on element feeder lengths, I have chosen 100° for these feeders (at 3.8 MHz, 47.45 feet), which is more than sufficient.

The input ends of the feeders to the two middle elements are paralleled and a shunt input L-match is used to transform this combined load directly to the desired pure resistance at the common connection to the array. The input voltage for this chain is the design objective for the networks feeding the remaining elements. Consequently, after transforming the other elements to their required pure resistances (equalling 50 ohms when all networks are paralleled), coax-equivalent π circuits are used to match the voltage phase required at the common connection of the array.

table 4. Network input parameters for a *triangular array*.

element 1	**element 2**	**element 3**
Z_1 $20.4 - j10$	Z_2 $78.4 + j4$	Z_3 $78.4 + j4$
I_1 $1\angle 0^\circ$	I_2 $0.5\angle -90^\circ$	I_3 $0.5\angle -90^\circ$
E_1 $22.72\angle -26.11^\circ$	E_2 $39.25\angle -87.08^\circ$	E_3 $39.25\angle -87.08^\circ$
90° Coax	**90° Coax**	**90° Coax**
Z_1 $98.81 + j48.43$	Z_2 $31.81 - j1.62$	Z_3 $31.81 - j1.62$
I_1 $0.45\angle 63.89^\circ$	I_2 $0.79\angle 2.92^\circ$	I_3 $0.79\angle 2.92^\circ$
E_1 $50\angle 90^\circ$	E_2 $25\angle 0^\circ$	E_3 $25\angle 0^\circ$
	elements 2 & 3 paralleled	
shunt input L	**shunt input L**	
Z_1 $146.08 + j0$	Z_2,Z_3 $76.02 + j0$	
I_1 $0.37\angle 29.22^\circ$	I_2,I_3 $0.72\angle 65.70^\circ$	
E_1 $54.59\angle 29.22^\circ$	E_2,E_3 $54.59\angle 65.70^\circ$	
	tee circuit	
	Z_2,Z_3 $76.02 + j0$	
	I_2,I_3 $0.72\angle 29.22^\circ$	
	E_2,E_3 $54.59\angle 29.22^\circ$	

coming soon

In a forthcoming article in this series I shall cover in detail a method of calculating simple 4-terminal networks based on matrix algebra. As with many mathematical procedures, application of the procedures to the solution of problems doesn't necessarily depend upon a complete understanding of the underlying theory. It is a tool simplifying what is otherwise a tedious process. The calculation procedures are structured (offering equations for matrix values), making it ideal for programmable calculators or small computers. For the same reason, the procedure easily lends itself to chained calculations, allowing fast analysis of a cascade of networks.

reference

1. Forrest Gehrke, K2BT, "Vertical Phased Arrays, Part 1," *ham radio*, May, 1983, page 18.
2. Forrest Gehrke, K2BT, "Vertical Phased Arrays, Part 2," *ham radio*, June, 1983, page 24.
3. Forrest Gehrke, K2BT, "Vertical Phased Arrays, Part 3," *ham radio*, July, 1983, page 26.
4. Roy Lewallen, W7EL, private communication. Contact K2BT for further information.
5. See "Short Circuits," *ham radio*, October, 1983.

ham radio

clean it up

Dear HR:

In his Ham Note, "Low Duty Cycle Transmitter Tune-up," (August, 1983), K4KI recommends using an automatic keyer in the dot mode during transmitter tune-up in order to reduce the duty cycle by approximately 50 percent and thereby save wear and tear on the tubes. But although an editor's note recommending the use of a dummy load at all times was included, we all know not every ham has a dummy load. Those who do have dummy loads don't always use them, and there are times when loading must be done into a radiating antenna, even if only briefly.

Therefore, K4KI's alternatives of either feeding the keyer's audio sidetone through the microphone, or feeding in audio generated by a relay connected in an RC time-constant circuit to make it buzz were both most unfortunate suggestions. Hopefully anyone currently using either of these techniques while loading into anything other than a dummy load will discontinue the practice.

While it is true that a pure sine wave carefully fed into the microphone input of an SSB transmitter can produce, for all practical purposes, a sine wave RF carrier in the output, this should never be attempted casually. The signal should be monitored on an oscilloscope, as the slightest distortion of the input sine wave will result in trash signals in the RF output which may vary from mild to an RF carrier output composed of numerous signals. Add in some flat-topping in the finals, and this garbage will extend perhaps several hundred kHz above and below the intended frequency of operation.

Few, if any, audio sidetones are pure sine waves. This is true whether it is electronic keyer sidetone, or the CW sidetone audio now included in all modern transceiver designs. At least one commercial electronic keyer I've seen uses a diode in series with the speaker, clipping one half of the audio waveform and generating a truly unique sidetone signal! Sidetone signals also tend to have an abundance of clicks or chirps, often both. Many of the CB to 10-meter conversions have no provision for operating CW if they are ex-CB SSB transceivers. It is common to feed keyed sidetone audio into them through either the mike or mike input circuit, and the result is spectacular. A number of these are currently loose on 10 meter CW and their signals are characterized by what can only be described as sounding like keyed steam calliopes — numerous carriers, usually accompanied by a bad case of chirps and/or clicks. A fairly clean one will occupy 5 or 10 kHz.

Audio sidetone-generated signals fed into the mike or mike input circuit will invariably generate unsanitary RF output from an SSB transmitter. Sending it into a dummy load is one thing, but radiating this garbage is quite another. Where on-the-air transmitter tuning is unavoidable, only the CW mode utilizing the transmitter's internal CW keying circuit and a keyer set for fast dots is appropriate. Hopefully the internal keying circuitry will provide proper shaping to avoid generating key clicks (approximately 5 ms rise and decay times), and if the dot/space ratio is correct, this will reduce the duty cycle to something less than 50 percent.

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short circuit

phased verticals

One line of K2BT's article, "Phased Vertical Arrays: part 4" (October, 1983) was inadvertently omitted. The second-to-last sentence on page 45 should read as follows:

"The calculation procedures are structured and identical for any circuit (except for the differing equations for matrix values), making the method ideal for programmable calculators or small computers."

vertical phased arrays: part 5

ABCD matrix parameters simplify network calculations

The most recent article of this series[1] discussed the application of lumped-constant circuits to the drive networks for all-driven element phased arrays. Design equations were presented for the most commonly used four-terminal networks. The design process and general procedures that must be followed for any drive network were reviewed, using typical arrays as examples.

Not discussed in part 4 was how the input/output calculations might be done. Those familiar with complex algebra and the use of a Smith chart can do these calculations one circuit branch at a time with these relatively simple networks. But it is a tedious process at best, prone to human error and cumulative errors resulting from rounding off in chain calculations. And one of Murphy's Laws asserts that errors are always committed at the *beginning* of the longest chain of calculations!

There is an alternative to this drudgery: matrix algebra. Using it allows us to determine the input conditions that result when any load is connected to a network. We do not have to calculate each circuit branch individually; all we need to know is the circuit type. There is even a built-in method of checking accuracy which aids in eliminating arithmetical errors and the entry of incorrect signs.

Matrix algebra has been used for solving problems of networks, transmission lines, and filters since the late 1940s because of its convenience in rapid circuit analysis and synthesis. With the advent of computers the use of matrices has increased, and it is possible that many readers have been using matrix methods without recognizing them as such, since the methods have often been incorporated into scientific computer programs as special process calls or library functions.

If matrix methods have been neglected in Amateur literature, it is not for lack of suitable applications. Perhaps the method has seemed too esoteric to be applied to such mundane problems as matching antennas to transmission lines, or that the jargon associated with it has frightened experimenters away; perhaps Amateurs have simply not been sufficiently exposed to this powerful mathematical tool.

four-terminal network matrices

Mathematical analyses using matrices require only the algebra of alternating current theory: a + jb. Because four-terminal networks happen to have properties which are natural to matrices and because the recurring structure of matrix operations makes them well suited for performance on programmable calculators or small computers, it makes sense for the Amateur to apply matrix algebra to network design.

By Forrest Gehrke, K2BT, 75 Crestview, Mountain Lakes, New Jersey 07046

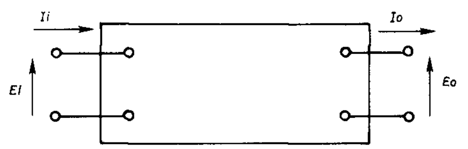


fig. 1. Four-terminal network showing reference directions for voltage and current. Subscripts i and o imply 'in' and 'out', respectively.

As with many other mathematical concepts, the application of matrix methods does not necessarily require a complete understanding of the underlying theory. It is in this context — i.e., in explanation of the *use* of matrices — that I address this subject. (For those who may wish to explore matrix algebra in depth, a brief bibliography is supplied at the end of this article.)[2]

Some fundamentals, such as the assumptions and restrictions and the notation employed, must be understood. Before all else, it should be emphasized that we are dealing only with alternating current *steady* state. (Matrix methods can be applied to transient and pulsed states, but that is outside the realm of this discussion.)

Four-terminal networks are a special form of a general network having a pair of input terminals and a pair of output terminals. Pictured in **fig. 1** is a box with the two pairs of terminals showing the reference directions for voltage and current. We may not know what is in the box, but we will suppose it to consist of any number of circuit branches and impedances with the following restrictions applying:

1. All impedances may be complex, but are linear and constant (time-invariant).

2. The network is passive, i.e., the only generators must be external sources (no dependent or internal sources), represented by E_i or E_o operating alone, or both simultaneously.

Despite not knowing the exact circuit inside the box, enough information can be deduced from measurements (amplitude *and* phase) made at the four terminals to produce a simple Tee or π circuit which is equivalent to it at any one frequency. We do this by defining a set of matrix parameters as follows:

$$A = (E_i/E_o) \qquad \text{with } I_o = 0 \tag{1}$$

Voltage E_i is applied to the input terminals and voltage E_o is measured at the output terminals with no load connected to the output.

$$B = (E_i/I_o) \qquad \text{with } E_o = 0 \tag{2}$$

Voltage E_i is applied to the input terminals and the short circuited output current I_o is measured.

$$C = (I_i/E_o) \qquad \text{with } I_o = 0 \tag{3}$$

Applying a voltage to the input terminals, measure input current I_i and output voltage E_o with no output load connected.

$$D = (I_i/I_o) \qquad \text{with } E_o = 0 \tag{4}$$

Applying a voltage to the input terminals, measure input current I_i and measure the short-circuited output current I_o.

The coefficients A, B, C, D are called general network parameters. Two relationships exist between inputs and outputs of a four-terminal network involving these parameters:

$$E_i = AE_o + BI_o \tag{5a}$$

$$I_i = CE_o + DI_o \tag{5b}$$

A and D are dimensionless transfer ratios, but B and C have the dimensions of impedance and admittance, respectively. In addition, a specific relationship exists between the network parameters because of reciprocity:

$$AD - BC = 1 \tag{6}$$

If we know any three of the four parameters, we can calculate the fourth. On the other hand, if we believe we know all four, this relationship gives us a means of verification, for if calculations using **eq. 6** do not hold, there must be an error.

The matrix used to describe four terminal networks is known as a square or network matrix. The network matrix is always portrayed this way:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \tag{7}$$

If the network contains no resistances, i.e., is lossless, A and D are real numbers and B and C are pure imaginary numbers (B and C carry the 'j' operator).*

Several kinds of matrix parameters such as S, H, and ABCD, have been developed for solving particular problems. Though one system is sometimes preferred over the other, it is possible to convert any of these parameter types to any other type, and to use *any* type of matrix parameter in the calculations discussed herein.

**Complex algebra*, *real* and *imaginary*, are mathematician's terms. For people working with electronics, these terms may unfortunately convey meanings which are not intended literally. The algebra, although different, *is not complicated*; the reactances resulting from inductances and capacitances *are neither unreal nor imaginary in effects*. However, the terms have been with us a long time and are here to stay.

Matrices may be manipulated — added, multiplied, inverted, reversed, partitioned — in many ways, but only according to special rules of procedure and order. For example, when four-terminal networks are cascaded, the effect may be calculated using matrix multiplication. If the individual matrix of each component network is known, the product is a new matrix of ABCD parameters for the overall network chain, allowing calculation of input/output relationships directly, end-to-end. If there is no need to determine the intermediate voltages and currents of the component networks, this is the way to go. (The order of this matrix multiplication is important. Obviously, there will be a great difference in results if the position of a 75-ohm 1/4 wave transformer followed by any length of 50-ohm line, is reversed!)

A four-terminal network is reversible if the A parameter is equal to the D parameter at all frequencies. This is another special case of the network matrix; we may reverse the connections to such a network without causing any change. Physically, a length of coaxial transmission line, a symmetrical Tee or π circuit are examples of such networks. The matrix of reversible networks appears like this:

$$\begin{bmatrix} A & B \\ C & A \end{bmatrix}$$

with $A = \pm\sqrt{1 + BC}$

When designing four-terminal networks, it is often necessary to express the current and voltage at the input side as a linear transformation of the current and voltage on the output side. For instance, with phased arrays we usually know what is wanted on the output side. What we need to know is the current and voltage required on the input side to produce those specific output conditions. Rearranging **eq. 5**, we can state several useful relationships. If each equation is divided by I_o we have:

$$E_i/I_o = (AE_o/I_o) + B$$

and $$I_i/I_o = (CE_o/I_o) + D$$

E_o/I_o describes an external load which I will call Z_a. (I have chosen to use the subscript 'a' to avoid confusion with the commonly used 'o' which refers to a transmission line characteristic impedance.)

If the first of the above equations is divided by the second, and Z_a substituted for E_o/I_o, we obtain:

$$E_i/I_i = Z_{in} = (AZ_a + B)/(CZ_a + D) \quad \textbf{(8)}$$

which defines the input impedance of the network in terms of the output load Z_a and the parameters of a network.

This leads to additional useful relationships:

$$E_o/E_i = Z_a/(AZ_a + B) \quad \textbf{(9)}$$

$$I_o/E_i = 1/(AZ_a + B) \quad \textbf{(10)}$$

$$I_o/I_i = 1/(CZ_a + D) \quad \textbf{(11)}$$

$$I_i = I_o(CZ_a + D) \quad \textbf{(12)}$$

$$E_i = I_o(AZ_a + B) \quad \textbf{(13)}$$

If we know the input impedance and want to calculate the load impedance:

$$Z_a = (DZ_{in} - B)/(A - CZ_{in}) \quad \textbf{(14)}$$

If the matrices of the fundamental types of four-terminal networks are known, along with the input or output impedance, all other network characteristics can be computed. Notice the recurrence of the terms $AZ_a + B$ and $CZ_a + D$ in the above relationships. If we program only these two calculations, we have substantially reduced the tedium of network calculations. (Most scientific calculators include the functions rectangular-to-polar and polar-to-rectangular, which takes care of most of the rest of the computations.*) Notice also that the ABCD parameters define the network operating characteristics, independent of the type of network. Therefore, if a length of coax and a π circuit or a Tee circuit have identical parameter values, the circuits will be exactly equivalent (even though the equations for calculating the parameters are different for each of these network types). Though not *all* types of networks may be transformed from one form to another, it is true often enough not to ignore the possibility. If true, use it to simplify your design.

Part 4 showed the basic building block four-terminal networks most useful to design of the drive networks for phased arrays. Presented in **table 1** are the parameter equations for each of those circuits, using the same notation. Note that this discussion is confined to the lossless case, since for low band frequencies losses are usually negligible. In the more general case, the procedure is still valid with loss terms introduced. However, it's not needed in this discussion.

using the ABCD parameters

Example calculations, which usually improve understanding, also help illustrate the versatility of matrix methods. I will first show the relatively simple case of a quarter-wave transformer and then proceed to design a real network for a 2-element array.

*For those familiar with Hewlett-Packard calculators and RPN, an SASE to the author will bring a 98-step program developed for the HP-19C which can calculate **eqs. 8** through **13** using complex algebra. Translation for other programmable H-P calculators should not be difficult.

The electrical length of a quarter-wave transformer is 90 degrees. If we are not interested in circuit components values, we do not need to know the frequency. What is required is the angular length and the characteristic impedance of the circuit. "Electrical length" (in any units of length) is always defined as the length under *matched* load conditions. But this does not imply that the current or voltage phase displacement at other than matched conditions is necessarily equal to the electrical length of the circuit.[3] The quarter-wave transformer is an exception; this consequently accounts for its great utility. As long as the load is a *pure* resistance, the current and voltage phase displacement is 90 degrees even though not matched. If our transformer is made from a 50-ohm transmission line, it has the following ABCD parameters:

$$A = \cos 90^\circ + j0 = 0 + j0$$

$$B = 0 + j50 \sin 90^\circ = 0 + j50$$

$$C = 0 + \left(\frac{j \sin 90^\circ}{50}\right) = 0 + j0.02$$

$$D = \cos 90^\circ + j0 = 0 + j0$$

Assume that the load is a pure resistance of $35 + j0$:

$$AZ_a + B = 0 + j50$$

$$CZ_a + D = 0 + j0.7$$

and $Z_{in} = (AZ_a + B)/(CZ_a + D)$
$= 71.4285 + j0 \text{ ohms}$

Though we already know the current phase displacement, we can also determine the current amplitude ratio, a factor often required in antenna array calculations:

$$I_o/I_1 = 1/(CZ_a + D) = 1.4285 \angle{-90^\circ}$$

Assuming the load current, I_o, to be $1 + j0$, the input voltage is $E_i = I_o\ (AZ_a + B) = 50\ \angle 90^\circ$, another value often needed in array network design.

In short, with **eqs. 8** through **13**, (in some cases assuming values for output current or voltage), we can find any of the input conditions using the ABCD parameters of the circuit.

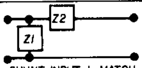
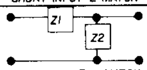
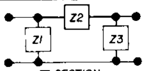
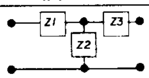
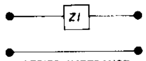
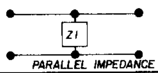
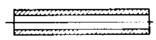
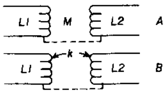
Suppose we had chosen a π network for our quarter-wave transformer. If we use a reversible π circuit designed to be coax-equivalent we know that[1]

$$X_1 = X_3 = -(Z_o \sin \theta)/(1 - \cos \theta)$$

and $X_2 = Z_o \sin \theta$

where Z_o and θ is defined the same way as it is for coax. As already indicated, the ABCD parameters define the circuit characteristics. Since the circuit characteristics are supposed to be the same, we should expect to find the parameter values to be identical even though calculated differently.

Table 1. Four-terminal network block diagrams and associated matrix forms.

No.	NETWORK	MATRIX = $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$
1	 SHUNT INPUT L-MATCH	$\begin{bmatrix} 1 & Z2 \\ 1/Z1 & (Z1-Z2)/Z1 \end{bmatrix}$
2	 SERIES INPUT L-MATCH	$\begin{bmatrix} (Z1+Z2)/Z2 & Z1 \\ 1/Z2 & 1 \end{bmatrix}$
3	 π SECTION	$\begin{bmatrix} \frac{(Z2+Z3)}{Z3} & Z2 \\ \frac{(Z1+Z2+Z3)}{Z1\ Z3} & \frac{(Z1+Z2)}{Z1} \end{bmatrix}$
4	 TEE SECTION	$\begin{bmatrix} \frac{(Z1+Z2)}{Z2} & \frac{(Z1Z2+Z1Z3+Z2Z3)}{Z2} \\ \frac{1}{Z2} & \frac{(Z2+Z3)}{Z2} \end{bmatrix}$
5	 SERIES IMPEDANCE	$\begin{bmatrix} 1 & Z1 \\ 0 & 1 \end{bmatrix}$
6	 PARALLEL IMPEDANCE	$\begin{bmatrix} 1 & 0 \\ \frac{1}{Z1} & 1 \end{bmatrix}$
7	 TRANSMISSION LINE	$\begin{bmatrix} \cos\theta & jZo \sin\theta \\ j\frac{\sin\theta}{Zo} & \cos\theta \end{bmatrix}$
8A 8B	 COUPLED CIRCUITS	$\begin{bmatrix} \frac{L1}{M} & \frac{j\omega(L1L2-M^2)}{M} \\ \frac{-1}{\omega M} & \frac{L2}{M} \end{bmatrix}$ A $\begin{bmatrix} \frac{1}{k}\sqrt{\frac{L1}{L2}} & j\omega\sqrt{L1\ L2}\left(\frac{1-k^2}{k}\right) \\ \frac{1}{j\omega k\sqrt{L1\ L2}} & \frac{1}{k}\sqrt{\frac{L2}{L1}} \end{bmatrix}$ B

$$A = \frac{0 + j\,50 \sin 90^\circ - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}}{0 - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}} = 0 + j0$$

$$B = 0 + j50 \sin 90^\circ = 0 + j50$$

$$C = \frac{0 - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ} + j\,50 \sin 90^\circ - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}}{\left(0 - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}\right)\left(0 - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}\right)} = 0 + j0.02$$

$$D = \frac{0 + j\,50 \sin 90^\circ - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}}{0 - j\,\frac{50 \sin 90^\circ}{1 - \cos 90^\circ}} = 0 + j0$$

Since the ABCD (parameter) values are identical, all other circuit relationships will be identical also. A similar parameter computation can be carried out with the coax-equivalent Tee circuit where[1]

$$X_2 = -Z_o/\sin\theta \text{ and } X_1 = X_3 = Z_o \frac{(1 - \cos\theta)}{\sin\theta}$$

Again the ABCD parameter values will be identical to the first two cases, though their computation requires yet another set of equations from **table 1**.

For the specific quarter-wavelength example we did not have to perform all these calculations to find the input impedance. We know all we need to know from the quarter-wavelength relationship,

$$Z_o = \sqrt{Z_{in} Z_a}$$

but when examining a new procedure it is always reassuring to be able to verify it with a more familiar one. Even for this example, we would not be able to calculate voltage or current input conditions quite so easily if the load were reactive. Best of all, matrix methods are applicable to any circuit without restrictions placed on electrical length. To illustrate this point, let's design a no-compromise feed network for a 2-element vertical phased array. Assuming directional switchability is desired, physical symmetry will dictate equal length element feeders. However, these need only be long enough to meet at a central switching point in the array. At a design frequency of 3.8 MHz, the array in this example has the following characteristics:

Equal amplitude current drive with a 90° phase displacement between elements. The elements are quarter-wave spaced and quarter-wave resonant.

From part 3 of this series[4] the driving-point impedances are:

$$Z_1 = 21.4 - j15 \text{ for element 1}$$

$$Z_2 = 51.4 + j15 \text{ for element 3}$$

Note: These impedances are for elements with an extensive ground plane.

At 3.8 MHz the element spacing is 64.71 feet. Allowing for some variation in placement of the switching relays and feed network, each feeder is arbitrarily cut to 34 feet. Assuming a line characteristic impedance, Z_o, of 50 ohms and velocity factor of 0.66 the electrical length of the feeders is 71.65°. The drive network will be matched to a 50-ohm line.

For a matched array we must first determine the resistance loads each network chain presents to the shack line. Assuming 1 ampere flowing into each element, the total power going to the array is the sum of I^2R inputs, or

$$21.4 \bullet 1^2 + (51.4) \bullet 1^2 = 72.8 \text{ watts}$$

table 2. Input conditions and ABCD parameters at each circuit junction with 1 ampere flowing into each element.

element 1		**element 2**	
Z_1	$= 21.4 - j15$	Z_2	$= 51.4 + j15$
E_1	$= 26.134 \angle -35.028^\circ$	E_2	$= 53.544 \angle -73.731^\circ$
I_1	$= 1 \angle 0^\circ$	I_2	$= 1 \angle -90^\circ$
71.65° coax		**71.65° coax**	
A	$= 0.3148 + j0$	A	$= 0.3148 + j0$
B	$= 0 + j47.458$	B	$= 0 + j47.458$
C	$= 0 + j0.01898$	C	$= 0 + j0.01898$
D	$= 0.3148 + j0$	D	$= 0.3148 + j0$
$AZ_z + B$	$= 6.7372 + j42.7351$	$AZ_a + B$	$= 16.1819 + j52.1798$
$CZ_a + D$	$= 0.5995 + j0.4062$	$CZ_a + D$	$= 0.0300 + j0.9757$
Z_1	$= 40.7999 + j43.6326$	Z_2	$= 54.9379 - j14.9217$
E_1	$= 43.2629 \angle 81.0410^\circ$	E_2	$= 54.6314 \angle -17.2296^\circ$
I_1	$= 0.7242 \angle 34.1195^\circ$	I_2	$= 0.9761 \angle -1.7656^\circ$
shunt L-match (leading)		**shunt L-match (lagging)**	
A	$= 1 + j0$	A	$= 1 + j0$
B	$= 0 - j116.2630$	B	$= 0 + j45.0951$
C	$= 0 - j0.01047$	C	$= 0 + j0.0079$
D	$= -0.21678 + j0$	D	$= 0.64378 + j0$
$AZ_a + B$	$= 40.7999 - j72.6303$	$AZ_a + B$	$= 53.9379 + j30.1733$
$CZ_a + D$	$= 0.2398 - j0.4270$	$CZ_a + D$	$= 0.7616 + j0.4260$
Z_1	$= 170.0934 + j0$	Z_2	$= 70.8171 + j0$
E_1	$= 60.3324 \angle -26.5554^\circ$	E_2	$= 60.3324 \angle 27.4573^\circ$
I_1	$= 0.3547 \angle -26.5554^\circ$	I_2	$= 0.8519 \angle 27.4573^\circ$
pi circuit (lag 54.0126°)			
A	$= 0.5876 + j0$		
B	$= 0 + j137.6306$		
C	$= 0 + j0.004757$		
D	$= 0.5876 + j0$		
$AZ_a + B$	$= 99.9479 + j137.6303$		
$CZ_a + D$	$= 0.5876 + j0.8091$		
Z_1	$= 170.0934 + j0$		
E_1	$= 60.3324 \angle 27.4572^\circ$		
I_1	$= 0.3547 \angle 27.4572^\circ$		

The voltage at the common connection of the array for matched conditions, i.e., a 50 ohm load, is $E = \sqrt{RW} = \sqrt{(50)(72.8)} = 60.3324$ *volts*. Since the element networks will be transformed individually to pure resistances whose paralleled value is 50 ohms, we must find the individual values. Going back to the resistive components of each driving-point impedance (and knowing the array's impressed voltage amplitude when correctly driven), we can determine what these resistive loads must be. Using the relationship $R = E^2/W$, these transformed loads are respectively:

$$(60.3324)^2/21.4 = 170.0935 \text{ ohms, element 1*}$$

$$(60.3324)^2/51.4 = 70.8171 \text{ ohms, element 2}$$

*Check whether these resistances when paralleled equal 50 ohms.

As recommended in part 4, the simplest network often results if the drive network for the $-90°$ phased element establishes the voltage amplitude and phase for the array common connection. Proceeding on that basis we will design the network for element 2 first. We need to transform the driving-point impedance of this element to the input end of its coax feeder and to determine the input voltage and current that must exist there. **Table 2** lists the input conditions for each element at each junction point and the ABCD parameters for each circuit. For simplicity, 1 ampere is assumed to be flowing into each element.

The design procedure for the lumped-constant part of the network transforms element 2 driving-point impedance, as seen at the input to its coax feeder, to the resistive load required, using a lagging phase shunt L-match. (The design equations for all discussed circuits are found in part 4.) This fixes the voltage amplitude and phase required for the array. Element 1 driving-point impedance, as seen at the input to its coax feeder, is transformed, with a leading phase shunt L-match, to the resistive load required for this chain. At the input to this L-match, the voltage amplitude is correct but the phase displacement has overshot the objective. (L-match circuits can be designed to produce either a specific resistive input or a specific phase displacement — not both.) The solution is to add a lagging π coax-equivalent circuit with a characteristic impedance of 170.0935 ohms and an electrical length equal to the difference between the phase angle existing at the input to this L-match $(-26.5554°)$ and the angle required at the common connection point $(27.4572°)$. This difference, i.e., the total angular displacement between these two vectors, is $54.0126°$. This phase correction circuit can be thought of as though we had somehow magically obtained approximately 26 feet of coax delay line having a characteristic impedance of 170 ohms. When doing chained network calculations, don't forget that Z_a (output load) for the circuit being computed is the *input* impedance of the preceding junction (looking towards the load). For example, when transforming the element driving-point impedance to the input end of its feeder, then the output load, Z_a, is the element's driving-point impedance. But when computing input/output relations for the succeeding L-match circuit, Z_a is now the impedance seen at the input end of the transmission line feeder, and so on.

final 2-element network design

The resulting design for the feed network of this array requires three inductances and four capacitances as seen in **fig. 2**. The component values are quite realizable. Evaluation of network designs is admittedly somewhat subjective. It is conditioned by the number of individual network circuits required, circuit component values (e.g., at 3.8 MHz lossless air core series arm inductances greater than 10 μH become physically large; shunt arm capacitance values less than 50 pF require more rigorous assessment of the unavoidable stray circuit capacitances). Should awkward values of components result from a particular design, it is often possible that using a different element to establish the voltage amplitude and phase at the array common connection point results in more physically realizable components.

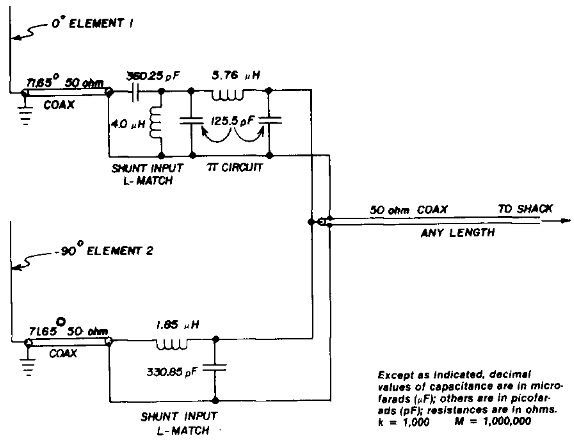


fig. 2. Feed network schematic for quarter-wave spaced two-element vertical array.

In a concluding article of this series I will discuss practical array and feed network construction and measurements.

This information, gleaned during the various phases of the development of my vertical phased arrays, should help the reader convert the theory presented on these pages into an actual physical structure — an antenna array that works the way it was designed to work.

references

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ham radio

vertical phased arrays: part 6

Building the array and measuring performance

In this final article of my series on vertical phased arrays I will discuss some of the practical aspects of putting up an array — how to build it, how to construct networks, and how to take measurements. I will also address a few questions readers have raised about my previous articles.

siting elements: with respect to the world

Situating elements by eye can be deceiving. Having said this, I am absolutely certain that some will try it, nevertheless. Hopefully, you will discover any errors before a large radial ground system has been installed. Unlike adjusting the elements of a rotatable Yagi, adjusting the spacing of a ground-mounted vertical phased array is a major undertaking that may require several weeks of effort. If you know the variation from true north that your magnetic compass tells you is north, fine. Otherwise, the best way is to line up with the north star, Polaris. This star is easy to locate; the outer two stars outlining the dipper of the Big Dipper form a pointer to Polaris. I have a 4-square array whose major lobes are turned off of the desired directions because I failed to determine the local magnetic variation. Sources for this information include your local airport, any office of the FAA, or persons associated with private aviation. Determine whether the variation is east or west. Generally, this variation will be *west* for those located east of a line running through Chicago and Miami and *east* if located west of that line. For example, at New York City the variation is approximately 12 degrees west. This means that true north for a magnetic compass pointing at north is 12 degrees rotated clockwise toward the east. This variation from true north slowly changes with time; if your information is more than 10 years old, find a more recent source.

Since most of these arrays have half-power beam widths of 90 degrees or more, why be so concerned over a few degrees? For forward gain small errors in pointing do not matter much; we are more interested in the directions in which the beam should *not* be pointing. Just as with Yagis, it is far easier to determine the *direction of nulls* than maxima. This is important diagnostic information: to the extent that these are in the directions and reduced with respect to forward gain as predicted, we have a reliable validation of the design.

siting elements: within the array

Accurately locating the elements of an array, particularly if they are not to be in line, isn't as easy as it might appear. Getting the correct angles is the problem. Euclid had the right idea; three points not in a straight line uniquely define any triangle. Using wire with little or no stretch (steel or aluminum fence wire is excellent), carefully measure out three lengths, each equal to a side of any triangle that outlines all or part of your array. Join the ends, and with two helpers, pull the wires taut, you'll have three points accurately located with respect to each other. If your array is triangular, you're all set. If it's a 4-square, you have only to locate the fourth element with the same wire triangle by turning it over on its diagonal. Triple-check

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table 1. Single 80-meter element tubing requirements.

quantity	length		diameter		wall		cumulative height	
3	10′	(3.05 m)	1-1/2″	(3.81 cm)	0.125″	(3.18 mm)	30′	(9.14 m)
1	10′	(3.05 m)	1-1/4″	(3.18 cm)	0.125″	(3.18 mm)	39′4-1/2″	(12.00 m)
1	8′	(2.44 m)	1″	(2.54 cm)	0.057″	(1.45 mm)	46′8″	(14.22 m)
1	8′	(2.44 m)	7/8″	(2.22 cm)	0.057″	(1.45 mm)	54′2″	(16.51 m)
1	8′	(2.44 m)	3/4″	(1.91 cm)	0.049″	(1.24 mm)	61′8″	(18.80 m)
1	4′	(1.22 m)	1/2″	(1.27 cm)	0.049″	(1.24 mm)	63′6″	(19.35 m)

Additional material requirements for a single element.

quantity	length		diameter		wall		
2	15″	(0.38 m)	1-1/4″	(3.18 cm)	0.125″	(3.18 mm)	mating inserts
1	24″	(0.61 m)	1-1/4″	(3.18 cm)	0.125″	(3.18 mm)	extender
1	18″	(0.46 m)	7/8″	(2.22 cm)	0.049″	(1.24 mm)	reinforcement
7	S.S. helical hose clamps approximately 2 inches (5.08 cm) OD						
9	S.S. 1/4″ - 20 1/2″ screws						
8	S.S. 8-32 1/2″ screws						
1	0.250″ (6.36 mm) female quick disconnect terminal						
1	0.250″ (6.36 mm) male quick disconnect terminal						
1	SO-238 UHF female terminal						
12″ (30.5 cm) flat tinned copper braid							
500′ (152.4 m) 1/8″ (3.18 mm) nylon woven cord							
7800′ (2377 m) PVC insulated No. 24 solid copper wire (100 0.3 wavelength radials)							

everything to be sure, because array element layout is one of the few *physical* items under your complete control among the factors determining array symmetry. In prior articles I showed that electronic beam switching requires every element to operate *identically* in each of the *different* electrical positions of the array. This is a severe requirement; the best we can hope for is to get within 5 percent of meeting it, realizing that reaching within 10 percent results in a significant loss in F/B performance.

For those who may want to check array patterns, I have observed that reception of a 1-watt signal source located between a 1/4 to 1-mile distance is consistent with the pattern that is seen at the vertical angle of maximum radiation (but without QSB). However, at 20 miles this is no longer true because high vertical angle reflections predominate, sometimes so strongly that a positive F/B is seen.

monopole construction

After much experimentation with a variety of ways to put together tubular quarter-wave length ground-mounted 80-meter vertical elements, I hit upon a method of construction which has held up for over six years. It's relatively inexpensive, but has withstood the rigors of northeastern winters, including icing followed by 80 MPH winds. After failures with lighter designs I decided that, at least for 80 meters, any tubular construction must be able to withstand being raised in one piece. If a vertical can withstand such stress, then it should also be able to survive high winds, icing, and even the temporary loss of one or two of its nine guys.

Table 1 lists dimensions of aluminum tubing that, when assembled into a quarter-wave element, will meet this criterion. Included with the table is a complete list of materials for a single element. If care is taken not to raise the antenna abruptly, it will stand tall and straight — despite all appearances to the contrary — as it is brought upright.

All tubing will telescope into its next larger diameter mating member except the lower 1 1/2 inch (3.81 cm) diameter lengths. For two of these lengths, a 15-inch (38.1 cm) section of 1 1/4 inch (3.18 cm) diameter 0.125 inch (3.18 mm) wall tubing is bolted (using three 1/4 - 20 screws) at one end with 7 1/2 inches (19.05 cm) protruding, forming a mating junction with the next lower identical diameter tubing. The 1-inch (2.54 cm) diameter length of tubing requires a 15-inch (38.1 cm) length of 7/8 inch (2.22 cm) diameter tubing to be inserted for its entire length at the lower end to act as reinforcement because of the abrupt change in wall thickness at this junction. All lighter tubing is drilled and tapped for stainless steel 8-32 screws at two places spaced about 5 inches (12.7 cm) apart, at junctions. This is necessary to prevent the development of intermittent continuity after a few months due to wind vibration. The tubing, having little weight in this part of the vertical, cannot be depended upon to maintain good contact by gravity.

This element will resonate at approximately 3800 kHz. Inevitably, multiple elements will not resonate at precisely the same frequency even though they are identical in physical length. For exact matching of resonant frequencies, a 2 foot (61 cm) length of the 1 1/4 inch (3.18 cm) diameter 0.125 inch (3.18 mm) wall tubing is used at the bottom of the vertical. This piece has tapped holes every 2 inches (5 cm) for a stainless steel 1/4 inch-20 screw, which determines the amount of its length that can be inserted into the bottom of the vertical. This may be adjusted as measurements dictate.

Flat braid [approximately 12 inches (30.5 cm)] is doubled, a 0.250 inch (6.36 mm) female quick disconnect terminal soldered at one end, and clamped to the bottom of the vertical with a helical hose clamp. I wrap PVC electrical tape around this to keep the doubled braid together. This makes a flexible, low inductance connection to the feeder. The coax termination is an SO-238 UHF female connector to which is soldered a male 0.250 inch (6.36 mm) quick disconnect terminal. The reason for these terminals will become quite obvious as measurements begin.

Glass bottles, corked to prevent accumulation of rain, may be used as standoff insulators for the verticals, since the necks happen to fit within the element base.

guy wires

Three sets of three guys, one set every 16 feet (4.88 m) from the base, are connected by two hose clamps at each attachment point. One clamp acts as a backstop for the clamp immediately above it, which clamps around the nylon guys. The nylon guy ends are tied with their own guy and also with one of the adjacent guys as additional insurance (falling tree branches can tear away the first tie but the fall, once arrested, seldom takes out the second tie). The attachment areas are waterproofed with PVC tape.

An element is raised by threading one of the three middle guys (usually made longer than those adjacent, specifically for this purpose) through a pulley which may be as low as 35 feet (10.7 m) from the ground. Since my array is among trees, I chose one to serve as a ginpole — which, of course, requires a real ginpole if you have no trees. Identify all guys with their ground anchor location, and lay them out so that no crossovers will be necessary later. During raising, the two remaining middle guys should be controlled by helpers to restrain the element from moving to the right or left, and as it arrives near the vertical position, to restrain it from continuing in the direction of the raising pulley. Don't forget to instruct your helpers in this latter point; more than one vertical has been successfully raised, only to continue unrestrained on its path to an inglorious end as it passes the upright position!

I've found that 1/8 inch (3 mm) diameter white woven nylon cord (sometimes called parachute shroud) is an economical, strong, long life material for guy stays. This is available at K-Mart stores in 50- and 100-foot (15 and 30 meter) lengths. I have some still in use after six years. The same cannot be said for polypropylene rope. Even 1/4 inch (6 mm) UV resistant material will fail in just two years.

radial systems

Installing radial systems is the dog work of building a low band array. It is also where the payoff — which too few Amateurs collect — is. There are two benefits to be gained with an extensive radial system: low losses and, more importantly, a low vertical radiation angle. But there's no free lunch: forget the loose talk you've heard on the bands about the benefit of water tables a foot below the surface, or being located over high conductivity earth. Pure water is a very good insulator, and most fresh water is too. And "high earth conductivity" is relative; it is *very* poor compared to the conductivity of copper. (For a perspective, see **table 2.**) Metal stakes in the ground at the base of your vertical give you good lightning protection, but *not* a good ground plane. Nor are there any rediscovered long-lost shortcuts; forty-eight radials raised a few feet above the ground won't provide any more efficiency than the same number on the ground. Undoubtedly, the best of all worlds would be an island surrounded by seawater, but for the near-field we'd *still* want an extensive copper radial system.

table 2. Conductivity comparisons.[1]

low conductivity earth	0.0005	mho/meter
high conductivity earth	0.03	mho/meter
sea water	5.00	mho/meter
copper	58,000,000.00	mho/meter

The best indicator of a good ground plane is how close the resistive component of the radiator's apparent self-impedance is to its theoretical resistance. The factors affecting theoretical radiation resistance are the electrical length and effective radius of the element, assuming a uniform cross-section monopole. Top hats, loading coils, and other means of shortening are also amenable to calculation, though the mathematics in some cases is more complicated.[2] For quarter-wave radiators this value is approximately 36 ohms.

In practice, there's another way to make this determination for any radiator without knowing the theoretical radiation resistance. It is the kind of analysis we usually wind up doing anyhow. Lay out radials, say ten at a time, distributed equally in all directions, taking measurements of the radiator's apparent self-impedance for each group of added radials. Plot these points on a graph as in **fig. 1** (open circuit any other elements of the array to avoid coupling). You will find that each lot of radials has less effect upon radiation

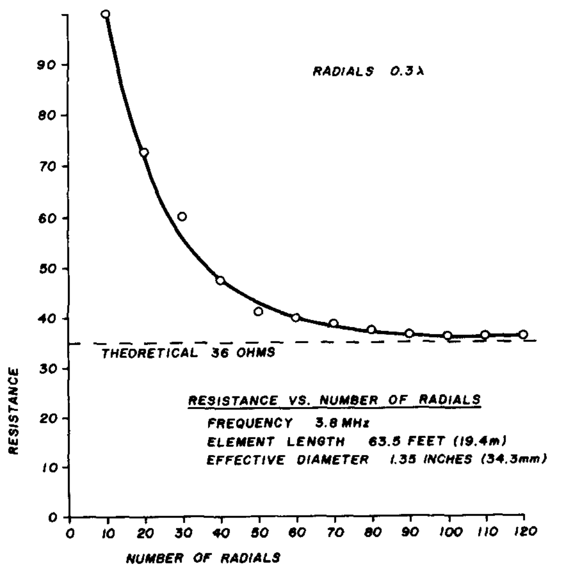


fig. 1. Input resistance of vertical antenna (includes all losses) versus number of radials.

resistance than the previous lot. Assuming radial lengths of a quarter-wave or more, after about 100 radials, the reduction in resistance with each added lot of radials is almost constant, but becomes vanishingly small (approximately 0.1 ohm). You'll notice that the curve on your graph has begun to flatten out; if you fit a French curve to this plot, you'll see that the plotted curve will nearly meet some horizontal line. In scientific terms, this is described as having become *asymptotic* with the theoretical resistance of the radiator; the horizontal line is a prediction of that theoretical value. (I am assuming negligible radiator element circuit losses. This is a fairly safe assumption for aluminum monopoles, but less safe if loading coils are present. More rigorously, the plot is becoming asymptotic with theoretical radiation resistance *plus* radiator circuit resistance.) Put another way, you've reached the point of diminishing returns. Although that point is self-definable, most experimenters would agree that it begins at the knee of that curve — i.e., at about *50* quarter-wave radials.

An aside to single vertical users accustomed to rating an antenna's merits according to VSWR readings: don't misinterpret an increase in VSWR as a negative indication when adding radials. Assuming VSWR is 1:1 with 50-ohm coax to a quarter-wave vertical, an appreciable ratio of output power (approximately 28 percent) is being used to heat the ground around the radiator. A higher VSWR after adding radials is desirable.

How should the radials be laid? This depends upon your personal aesthetics and also upon how the area occupied by the array is used. If you need to bury the radials (see "Build a Simple Wire Plow," page 107 — **Editor**), some form of protection against corrosion, such as PVC insulation or enamel coating, is necessary. Don't bury them too deeply; the closer radials are to the surface, the more effective they are. Some Amateurs have laid them flat on the surface and let grass cover them so well that limited traffic and even lawnmowers can be allowed.

What about wire size? I often hear people talk about laying No. 6 or No. 8 BS gauge radials. Unless you have to protect your system from farm animal traffic, this is calling for a hawser when thread will do. Consider: 1000 watts output to a single vertical with 25 radials (and assuming an apparent radiation resistance of 50 ohms). Each radial will be carrying all of 179 milliamperes, and *that* only near the base of the vertical.[3] Given a reasonable number of radials, a base current measured in amperes is divided into individual radial currents of milliamperes, and even this small current rapidly decreases as we move away from the base of the radiator. In the absence of concern about possible fragility, the wire size may be quite small.

Many articles in Amateur publications have suggested using steel fence wire or steel mesh as an economical substitute for copper. *Don't.* Unless the material has cost you nothing, and your labor is worth nothing, and you plan to abandon the antenna in less than a year, forget it. In just a matter of months, galvanizing — if present at all — is penetrated and corrosion proceeds. Steel, being magnetic, has a high permeability, making for a skin effect much thinner than copper when carrying RF current. Iron oxides are lossy semi-conductors. The thin skin effect, combined with a lossy surface, results in a wire which conducts *nearly zero* RF current long before it fails to conduct DC or has lost physical integrity — which takes only about three years after installation, and much less time if the system is buried. My first radial system consisted of a combination of aluminum, steel, and copper radials. It took a couple of years to solve the mystery of a slow but continually rising self-impedance of some of the elements in the array. In my efforts to overcome this rise, I compounded the mystery by adding more radials, which at first were — *more steel radials!*

array operation measurements

So you've constructed an array. Now you'd like to see how well it works. On-the-air tests are understandably at the top of your list. Unfortunately, this is not likely to be a good proof test of proper drive conditions, primarily because these arrays *want* to work and may show fair performance despite being poorly driven. This is almost always true for gain characteristics, and during some propagation conditions may even apply to F/B. So continue these tests, but give some thought to an old antenna man's advice, said to have been first enunciated during the period of Maunder's Minimum: "One swallowe prouveth not that summer is neare."*

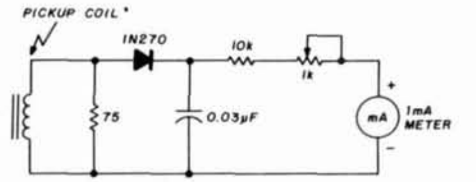
A much more definitive test is a measurement of element currents in each of the array directions. Assuming you have designed the feed network for a 1:1 VSWR, then element base current amplitudes measured within ±5 percent of design values and an array VSWR *no greater than* 1.15:1 in any direction is almost complete proof that drive is in the proper range, including current phase angles. Measurement of element current amplitude *and* phase is, of course, the ultimate test for drive conditons. A wideband dual-trace CRT is needed for this test, and since this equipment is quite expensive, may be beyond the reach of most Amateurs unless it can be borrowed. On the chance you have access to this equipment, a method is described in the next section of this article.

Measurement of current amplitude, a must for any serious array builder, is quite easy to do. At the lower frequencies a high degree of absolute accuracy is not difficult to achieve, but good linearity is really all that is necessary. For example, if actual current is doubled (power multiplied by 4), does the reading double? For this purpose the meter readings might just as well go from 1 ampere to 2 amperes, or 400 mA to 800 mA; we are more interested in good linearity of readings than in absolute value because phased array design considerations are concerned only with element current amplitude ratios.

Fig. 2 shows a schematic of an RF ammeter (**photo 1**). The basic meter movement may be anything up to 1 milliampere. I use germanium diodes for the rectifier because of their low turn-on voltage. This simple design works well for absolute accuracy and linearity up to 14 MHz. Low capacitance of the pickup coil to RF line is important and is increasingly so as frequency is raised. For this reason we want a high permeability factor for the toroidal core (for least number of coil turns). A Faraday shield will provide even more isolation, but this additional protection is not necessary at lower frequencies.

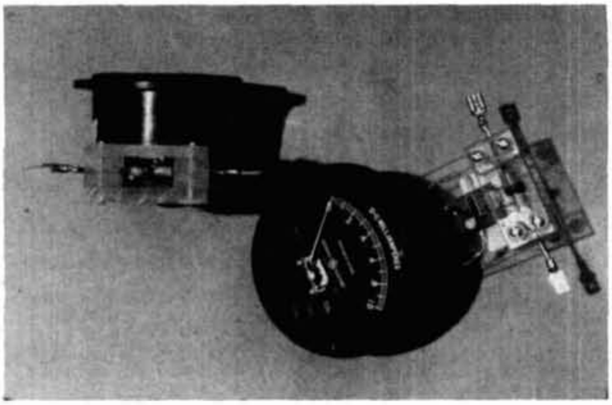
Since this ammeter is easy to duplicate, you may find it useful to have one for each element of the array because the efficiency of data collection is considerably improved. Note the use of quick disconnects for element terminals and associated measurement devices.

*From 1645 to 1715 there were no observable sunspots, and no Northern Lights. (Imagine a 70-year period in which the 10-meter band never opens and the 20-meter band is only so-so during the day, and dead at night!) A British astronomer, E.W. Maunder, in 1895, was the first to call attention to this strange behavior of the sun.



F37 Q1 FERRITE CORE
8 TURNS NO. 28 B.S. ENAMELED WIRE-EQUALLY SPACED
CORE CENTERED OVER NO. 10 CURRENT CARRYING WIRE

fig. 2. RF current probe circuit midscale reading corresponds to 1 ampere.



Solidly constructed RF ammeter uses quick connect/disconnect terminals.

dual-trace CRT measurements

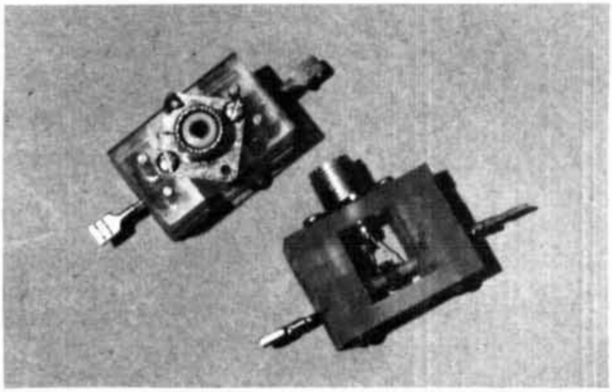
Measurement of RF current phase angles involves an instantaneous comparison of sinusoidal currents at the bases of the elements. Since the elements are widely separated physically (and distance is proportional to phase), we must take special precautions to be sure we are really observing events in time coincidence. One way is to make these observations at another location of our own choosing in such a manner that all events have been equally delayed. Though we will see events at some time later than they occurred, they will be in time coincidence. Identical coax lines will meet this requirement nicely. Obviously the pickup coils for sampling base currents also must be alike, with the further proviso that the terminations of these lines must be alike, resistive, and match the characteristic impedance of the lines. (Most dual-trace CRTs provide 50-ohm inputs.) The line length chosen should be long enough to allow measurements of the most widely separated elements. I am sure you've anticipated my comment that the assurance of identical electrical length is not provided by a tape measure. Furthermore, to help ensure line identity for other characteristics, it would be a good idea to cut these lines from a single piece of cable. (For an enlightening discussion on the variations that may be found in coaxial cable, see Bill Orr's "Ham Radio Techniques" in the January, 1984, issue.)

Photo 2 shows the construction of a pickup coil fixture. This is essentially the same as that used for an ammeter, but without the rectifier and filtering circuit. More attention must be paid to ensure that the pickup circuits have identical RF characteristics, however. We can test this by connecting both pickups in series as sensors in the same RF current circuit, preferably a resistive load. The amplitude and phase of the displayed current waveforms should be identical and should remain so when positions of pickups are interchanged. If reasonable care has been taken, the phase difference due to pickup coils should not exceed 2 or 3 degrees. Any small difference may be corrected for *a particular band* by connecting a mica capacitor (5-25 pF) across that pickup coil which *lags* in phase. My guess is that toroid core material variations are the probable cause of any slight differences. It is possible that substitution of another sample of the same core might also work; I did not investigate this.

In my experience with this measurement technique, I found it to be the most productive method for fine tuning a feed network to get the last bit of F/B performance improvement. Because both phase and amplitude changes are displayed, a much more rapid and intelligent analysis of cause and effect is possible. This comment applies with ever-increasing emphasis with the number of elements in the array. Lacking this capability, my advice is, "If it ain't broke, don't fix it!"; this can be as frustrating as attempting to adjust the color matrix board of a TV receiver without a crosshatch generator. The fact is that if self and mutual impedances are accurately read and used for the design and careful construction of the feed network, the array will be operating very close to, if not exactly at, optimum. The few adjustments determined with the dual-trace CRT are surprisingly miniscule "tweaks." Although the effect on F/B can be quite marked, for example, improving F/B from -20 dB to -30 dB, don't be carried away by these numbers: the effective frequency range over which this occurs is extremely narrow.

alternate methods of phase measurement

One would expect that considering its importance in antenna applications, measurement of phase angles at lower RF frequencies would have received more attention in the Amateur press than it has. A survey of Amateur publications did not yield much except one very interesting article directly applicable to phased array applications.[4] While the author's concept is ingenious and well chosen, it used a differential phase



Pickup coils provide signals for dual trace scope.

angle readout that was analog rather than digital. Considering the tremendous advances in semiconductor technology during the ten years since the article appeared, a digital readout should be possible. I hope that I may interest some enterprising experimenter to take up this challenge.

network construction

Several readers have commented that although the no-compromise advantage of 4-terminal feed networks is obvious, and that matrix algebra for the modular design of the networks is a powerful tool, they had apprehensions about how to turn the mathematics into working hardware. Some, familiar with the Pi networks seen in linear amplifiers, were discouraged by visions of the need for the same size of components and the cost of construction. Still others thought the networks might be difficult to tune.

None of these concerns are justified. Construction is actually quite simple, and with an accurate noise bridge,[5] tuning is easy. Tuning with an impedance bridge is done in the same step-by-step manner as is the design, and one of the prime advantages of this method is that it allows tuning to be done at the design frequency. The impedance levels of these networks are low, being generally in the 35- to 125-ohm range. Since each network chain is designed to appear *resistive* at its input and deals with only a portion of the transmitter power, voltages are seldom above 300 volts, even when driven by kilowatt linears. For example, I use postage stamp size mica capacitors extensively (500 volt rating) and I've yet to have one fail. Where high impedances are encountered, for instance, with elements requiring very little direct drive because of drive coupled from other elements, the current is so low that high voltage is not developed.

Photo 3 illustrates the simplicity and small component sizes. This is the feed network for my 80-meter 4-square array.[6] It is built into a 3 × 6 × 8 inch (7.6 × 15.2 × 20.32 cm) box on PC board, with each network chain individually removable. This takes the place of other feed methods which require 130 feet (39 meters) or more of coax for this band. At today's prices for good quality 50-ohm coax, this alone should be the deciding factor, without even considering the superior technical merits of 4-terminal feed networks.

As can be seen from the photograph, small 100 pF air variable capacitors are used as trimmers. Mica capacitors, singly or paralleled, using their color coded values, are chosen to make the required network capacitance fall in the middle of the trimmer range. All inductors carrying significant current (and these tend to fall between 0.5 μH to 5 μH) are air core using No. 10 or No. 12 B.S. enamelled copper. I wind these on 1-inch (2.54 cm) diameter wooden dowels, letting them spring up to a slightly increased diameter. Inductances significantly above 5 μH are wound on powdered iron toroids using No. 18 B.S. wire. Using single layer charts for the air wound coils or toroid core manufacturer charts, all inductors are wound to be well above the inductance required. A grid dip meter, together with a known capacitance, is used to trim the inductors to slightly above the required values (5 to 10 percent). The network is constructed with these components and is completed with the exception that no network interconnections are made nor are any connections made to the *shack line* coax terminal.

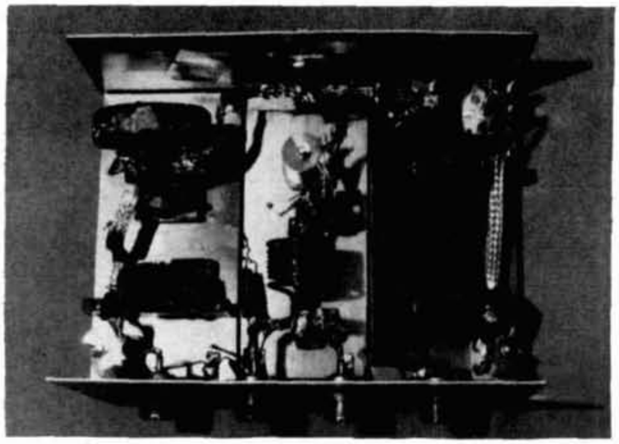
tune-up procedure

We have to choose which direction the impedance bridge will look into the network for tuning it. Each network chain was designed to transform a complex impedance to a pure resistance. Since it is much easier to duplicate resistances than complex impedances, this usually determines the choice. Assuming this case, consider a cascaded network consisting of a shunt L-match followed by a Pi circuit, which is the typical chain from the feeder of the −180 degree phased element of a 4-square array. For example, from Part 4[6] of this series, the input impedances as seen at the various points of a network chain are reproduced for element No. 4 of a 4-square array:

element No. 4 driving-point impedance	63.4 + j47.5
input impedance to 100 degree length feeder	22.73 − j11.37
input impedance to L-match	114.83 + j0
input impedance to Pi circuit	114.83 + j0

You will recall the reason for the Pi circuit was for phase matching, not impedance matching; therefore, its input impedance is the same as the impedance to the shunt L-match.

The impedance bridge unknown terminal is connected to the coax terminal, to which the element feeder would normally be connected. The resistor simulating the input resistance of the L-match (115 ohms is ok) is temporarily soldered to the input of the L-match at the shunt arm and ground (i.e., at the con-



Matching network for four square vertical array measures 3 × 6 × 8 inches (7.6 × 15.2 × 20.32 cm).

nection point normally going to the Pi circuit). Since we have chosen to measure conditions in the *reverse* direction through this network, we must consider what we want the bridge to "see" and set it accordingly. At first glance, it might seem reasonable to expect this to be equivalent to the impedance as seen at the input to the element feeder. Not so; this is not a bilateral case. Instead we should expect to see the *conjugate* of this impedance: i.e., 22.73 + j11.37. If the impedance bridge reads parallel circuit equivalents, these must be calculated and the bridge set accordingly. If the reactance is beyond the basic bridge range, an appropriate extender must be used. The tuning procedure is simplicity itself; without touching the bridge settings, adjust the L-match shunt trimmer for minimum detector output (being sure this minimum is within the trimmer range). If this is the normal impedance bridge null, the adjustment is complete. More likely, it is merely a minimum. Begin spreading out one of the outer turns of the inductor and readjust the trimmer. Since the null is sharp and deep, use care in spreading coil turns to be sure you have not passed through the null. (I use the tapered end of a pencil for this.) When the L-match is tuned, move the simulating resistor to the input of the Pi circuit (the point normally connected to the shack coax line terminal). Install an interconnection between the L-match and Pi circuit. With the bridge remaining connected and set as before, adjust the Pi circuit trimmers for minimum detector output and then reduce inductance by turn spreading. Since the Pi circuit has three interdependent adjustments, be sure to recheck the other two with each tuning change. The two trimmers should end up in approximately the same part of their range, assuming the fixed padders are similar. Since the tune-up of the Pi circuit is done with the bridge looking into the L-match, a separate procedure for integrating the two networks is unnecessary. Remove the resistor, but

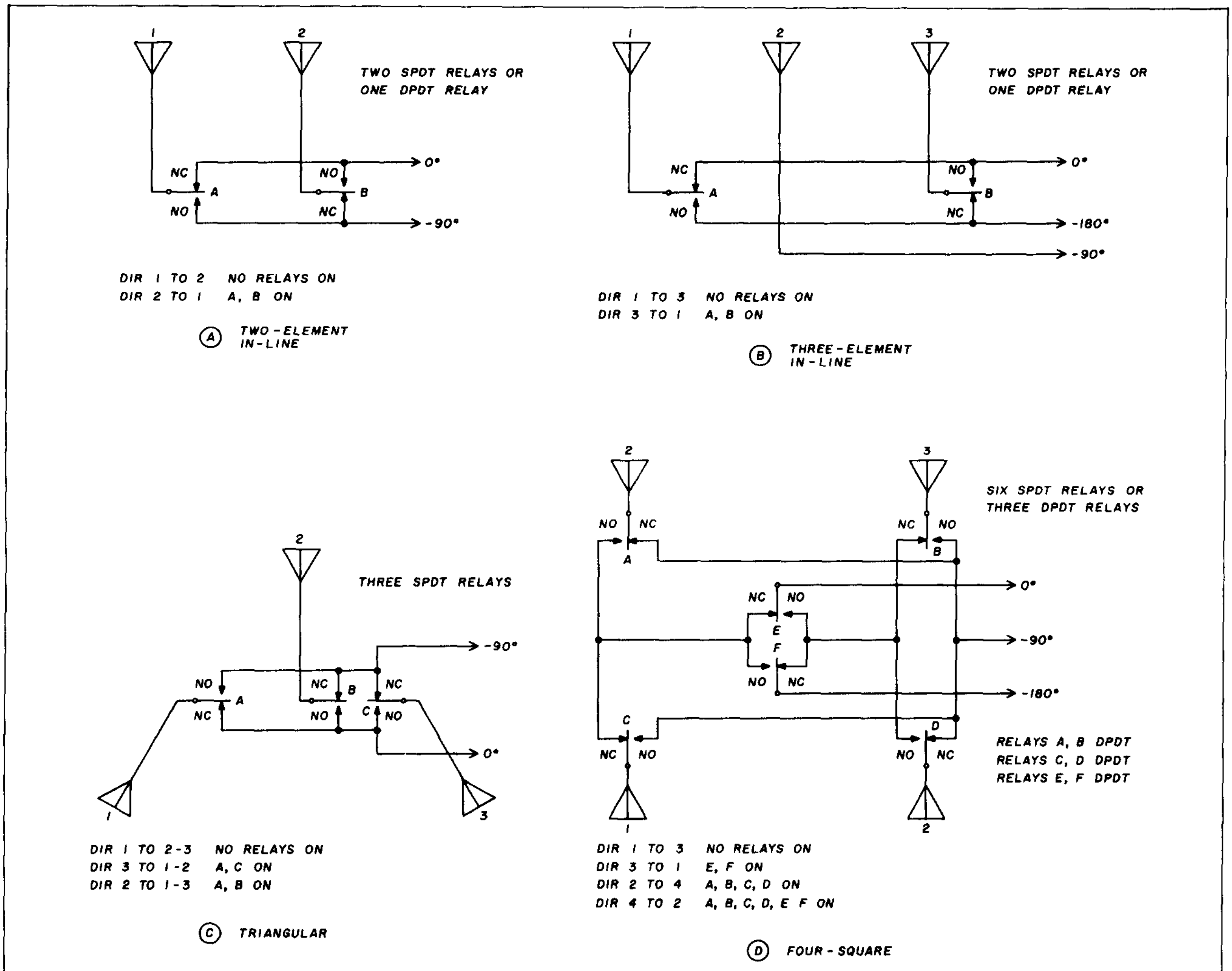


fig. 3. Relay interconnection diagram for four different vertical antenna arrays: (A) two-element in-line; (B) three-element in-line; (C) triangular configuration and (D) "4-square" configuration.

do not connect this chain to the shack line coax terminal until completing the adjustment of all chains.

In some circumstances it may be easier to simulate the calculated termination impedance of a network, in which case the bridge will look into this network in the same direction as the transmitter would. The impedance bridge is set to the pure resistance expected at the network input and it is connected to the shack line terminal. The simulated impedance (the same as calculated, not the *conjugate*) is connected to the coax terminal where the element feeder is normally connected. Assuming we are tuning the same network chain as above, a temporary connection of the shack line coax terminal is made to the input of the L-match (the same point at which the resistor was connected in the previous case). Except for these differences, the tuning procedure is the same. After tuning the L-match, the temporary bridge connection is transferred to the input of the Pi circuit, and an interconnection is made between the two networks in readiness for tuning the Pi circuit. After completion of tuning the Pi circuit, remove the shack line connection to it in preparation for tuning the next chain. Incidentally, there is nothing wrong with connecting the feeder coax into the chain to check out the entire network chain, making the appropriate changes to the bridge settings and/or the simulated network loads. However, do not connect an actual array element to this feeder in the expectation the element will present its array drive-point impedance, saving you the bother of simulating it. This simply won't work; part 4 of this series (October, 1983) explains why it will not.

directional switches

Relay interconnection diagrams for directionally switching four different vertical arrays is provided by **fig. 3**. When selecting relays for this application, remember that no one relay is switching *all* of your transmitter power; consequently ratings may be safely reduced. Since RF is being handled, ceramic insulation is advised, though I found no problem with linen bakelite at 80 meters. Always avoid "hot" switching relays. Even if they can stand it, your linear will not — and neither will the network, since high voltages will be present during switching.

Photo 4 shows a 4-square array relay construction that use three small telephone-type DPDT military surplus relays. At first I lost several relays each summer due to sympathetic discharges from lightning which would burn out the solenoids. This was cured by connecting a silicon high current diode in reversed direction across the coil in parallel with a 0.1 μF ceramic disc capacitor. I have since lost a few diodes to these discharges, but no more relays. Failed diodes are "shorts," so the 28 VDC supply to this system requires a protective series resistance to guard against this possiblity and to prevent damage to the power supply from the discharge.

on rounding-off calculations

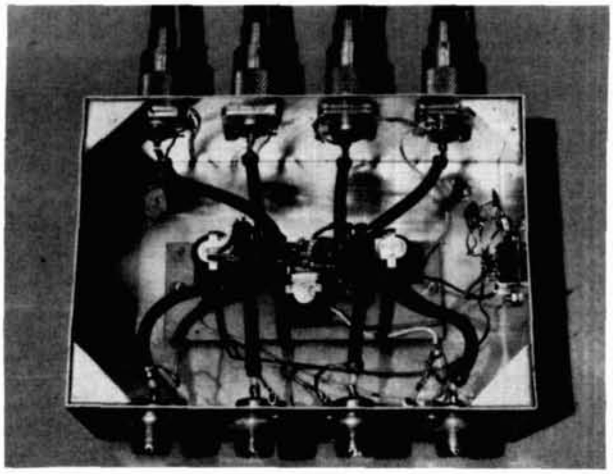
Calculator algorithms and computer operating system programs use guard digits as a means of retricting degradation of accuracy due to round-off in repetitive calculations. All values begin with and are calculated to one or two digits more than shown to the user. For example, if a calculator displays ten significant digits, it actually keeps values to eleven or twelve digits in its internal registers. Round-off errors thus tend to be restricted to these extra figures and seldom affect anything more than the least significant displayed digit. Since these extra digits rarely convey additional accuracy, they aren't displayed.

This concept also applies to calculations done by hand or with a slide rule. For example, using 3.14 for Pi reduces accuracy to only three significant figures *before* any calculations are done. A few computations immediately reduce that accuracy. Let's watch what happens with a simple calculation:

(Pi × 5)² − (Pi × 78)
= 1.57 if 3.14 is substituted for Pi.
If 3.1416 is used the result is 1.6965.
If 3.141593 then the result is 1.69591.

Note that the first approximation for Pi, accurate to three significant figures, has produced results accurate to only a *single* significant figure in just a few computations. What is happening is that every time this rounded off value for Pi is used, a small error is reintroduced, and in effect, the error is compounded.

Although many calculators round off to the nearest decimal, most small computers, and many large ones too, merely truncate values to some number of digits without adjustment to the nearest decimal, causing even more rapid divergence from accuracy if truncation is occurring after only a few digits.



3 × 5 × 7 inch (7.6 × 12.7 × 17.8 cm) box houses all relays for switching four square vertical array.

The point of this discussion is to convince you to keep computations to several significant digits more than the accuracy you'd like to end up with. Except in determining rough approximations, constants such as 3.14 for Pi, or the number 984, expressing the speed of light in millions of feet per second, should always have two or more additional significant digits. This becomes particularly important when trigonometric functions involving angles approaching quadrant boundaries (for example, 0, 90, 180, or 270 degrees) are being used. Anyhow, in this day of ubiquitous calculators and computers, calculation to ten significant figures represents no personal mental effort.

The following constants, important to many calculations, are given to a level of precision more than sufficient for most applications:

Pi	3.141592654
e	2.718281828 Naperian logarithm base
c	299.792456 × 10⁶ velocity of light, meters/second 983.571049 × 10⁶ velocity of light, feet/second
cm/in	2.54 (exact) Metric to English unit conversion

Readers have inquired about the values given in these articles for inductances and capacitors in 4-terminal networks. Where, for instance, is a capacitor of value 734.8 pF to be obtained? Obviously no capacitor of that value will be listed in any catalogue; neither could we hope to find it to such accuracy by a measurement and selection process without also controlling temperature, humidity, aging, and so on. Measuring a capacitor or an inductance to just *three* places requires careful technique. I showed values to greater precision in an effort to prevent (mostly unsuccessfully) confusion caused by round-off errors if readers attempted to work backwards from my values for a capacitance or inductance to compute reactances, voltages and currents.

The author, beside his modest station, has proven that over 260 countries can be worked on 75 meters with legal power and a good antenna system.

concluding comments

I thoroughly enjoyed putting this series together, even though it required far more time and effort than I could possibly have imagined. I hope it proves useful and educational, though I'm not sure whether the author or his readers gained more! I tried to leaven the theoretical with the practical, well aware of the difficulties and pitfalls of doing so in such a technical subject.

I introduced the topic of matrix algebra as a tool of nearly limitless versatility that literally begs to be used. It not only reduces the tedium of network design calculations and simplifies transformation of one network to another, but also makes child's play out of the calculations of input/output conditions when networks are cascaded. It is particularly well suited to computer programmed calculations because the fundamental algorithms are unchanging; only the specific network parameter calculations differ. I did not begin to plumb the possibilities in these articles; there are ABCD parameters specific to lattices (bridge circuits), bridged Tee's, all types of transformers, real coax (with loss) and on and on. I sincerely hope this alone has found fertile imaginations in which to take root. It mystifies me that so powerful a tool has found so little welcome in our engineering educational institutions.

Antenna experimentation has always been of absorbing interest to Radio Amateurs, whether for DXing, for propagation studies, superior contesting, or to satisfy one's curiosity. Even though we have the ability, most of us don't have the time to devote to exploring the complexities of our station equipment. But anyone can innovate with a piece of wire, and would that it will always be so. However, antenna experimentation isn't magic; it's a technology like any other. Most of the fundamental principles were established two generations ago, though many of the pioneers, whom we find referenced and footnoted in articles and texts, are still with us.

To Bob Booth, WB6SXV, and Mason Logan, K4MT, for their encouragement, advice, and careful proofreading — which more than once kept me honest — much deserved words of appreciation. Finally, I want to thank you, my readers, for the many kind comments you sent my way via letters and on the air, and most of all — for your patient attention.

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